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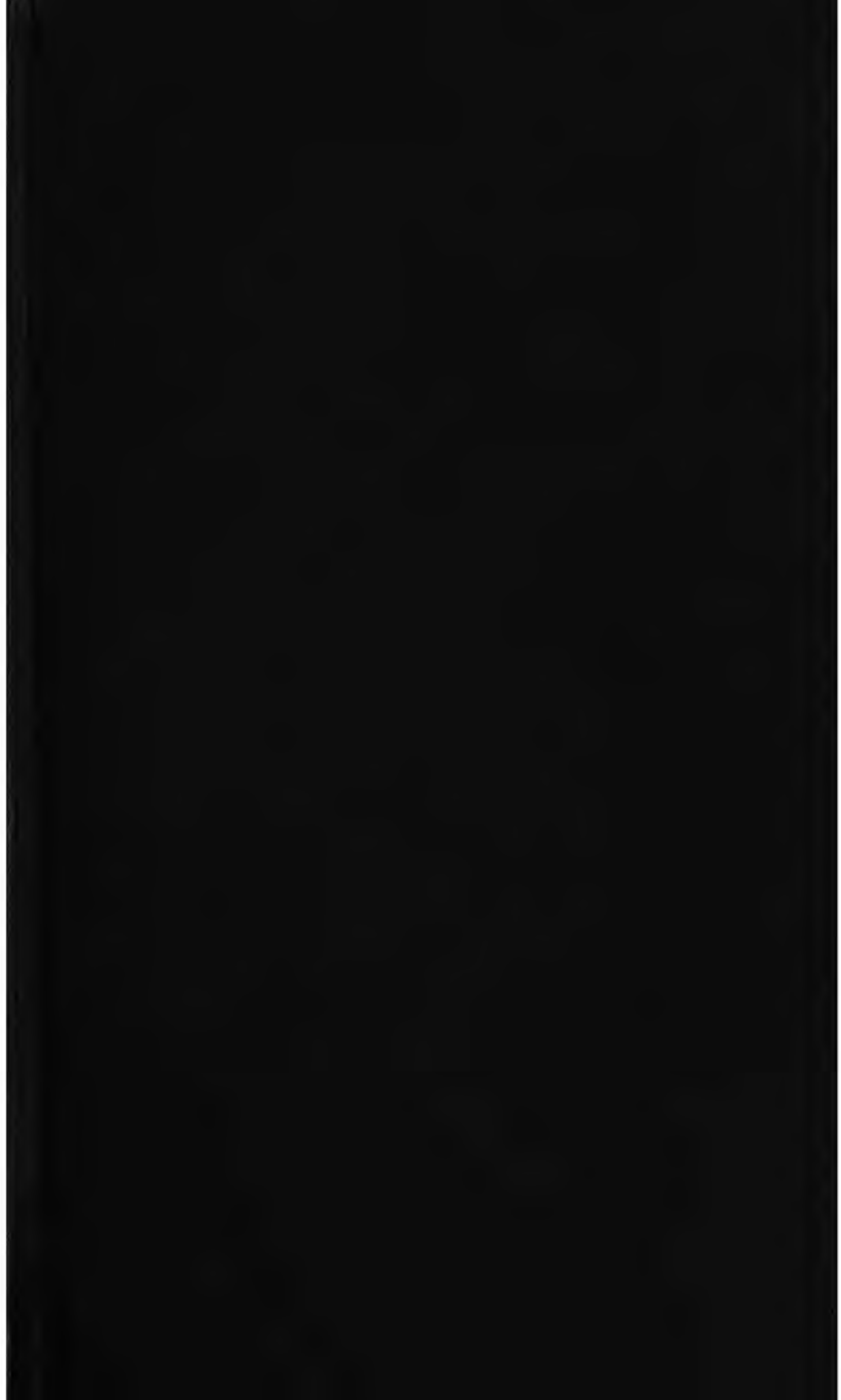
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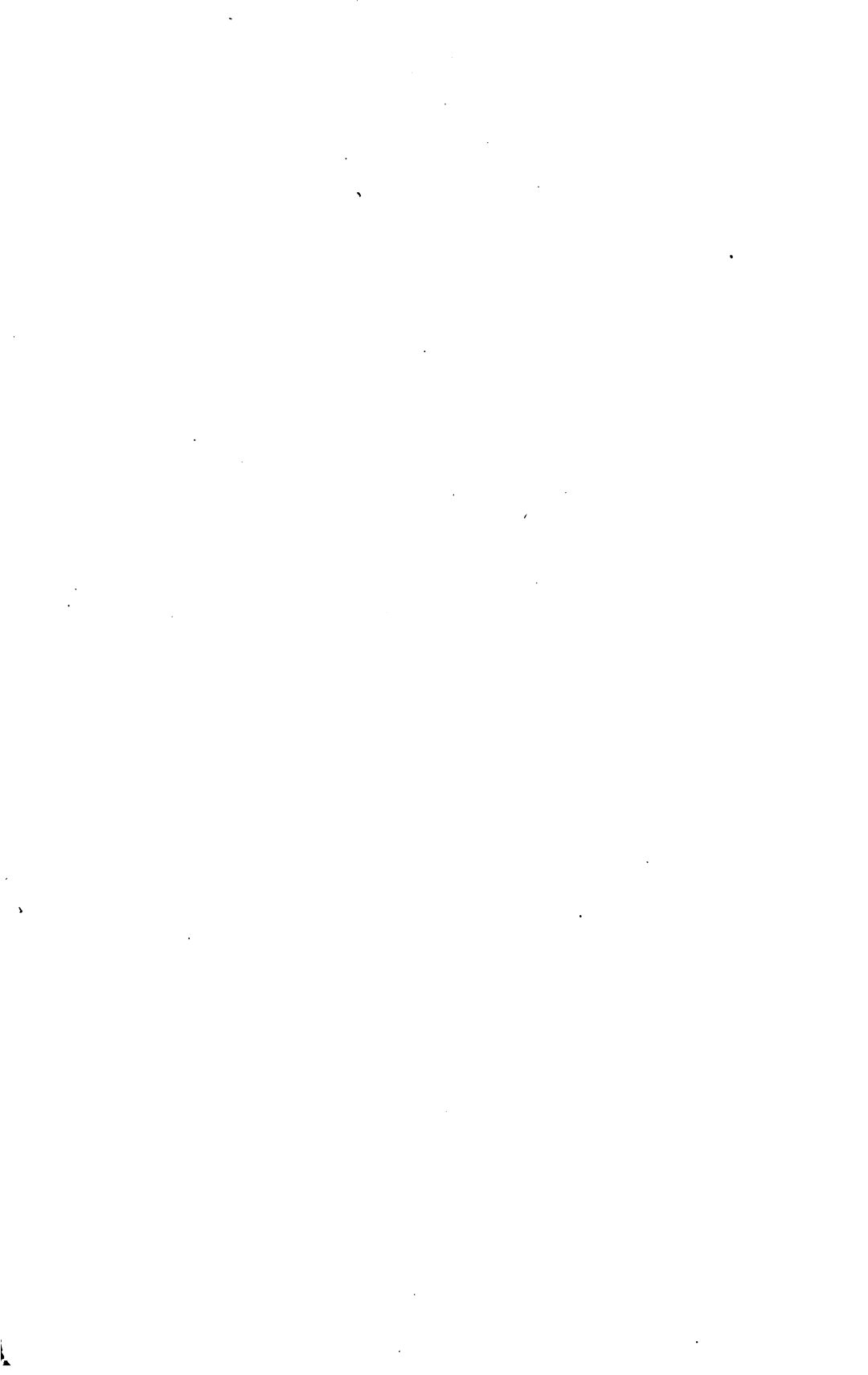
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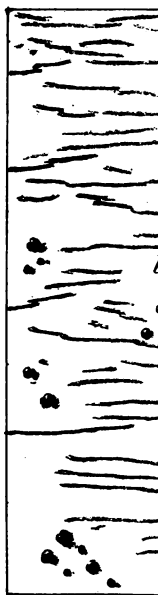
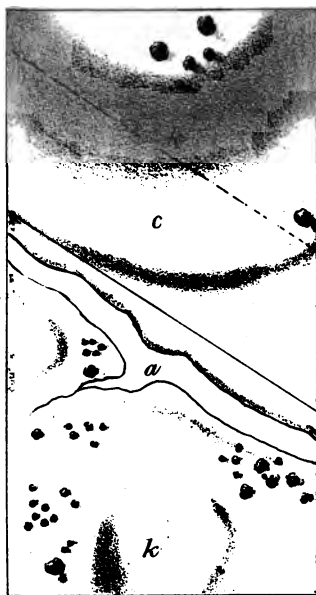
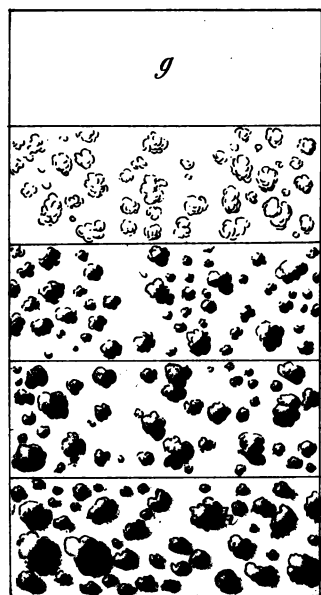
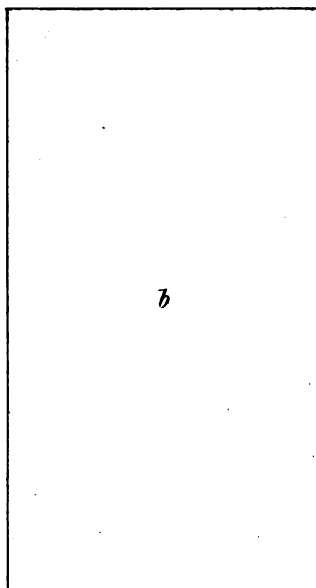
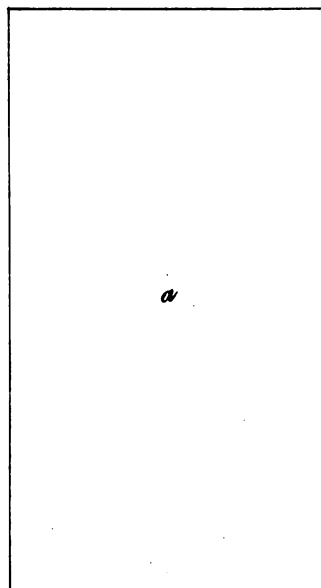
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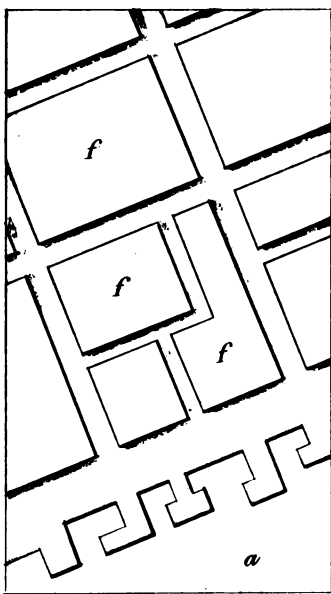
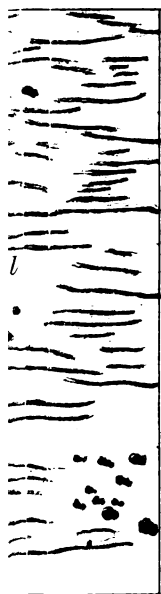
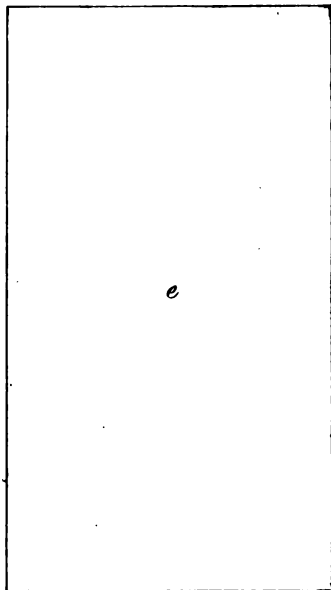
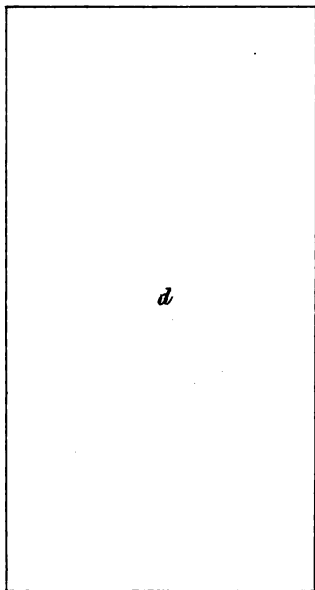
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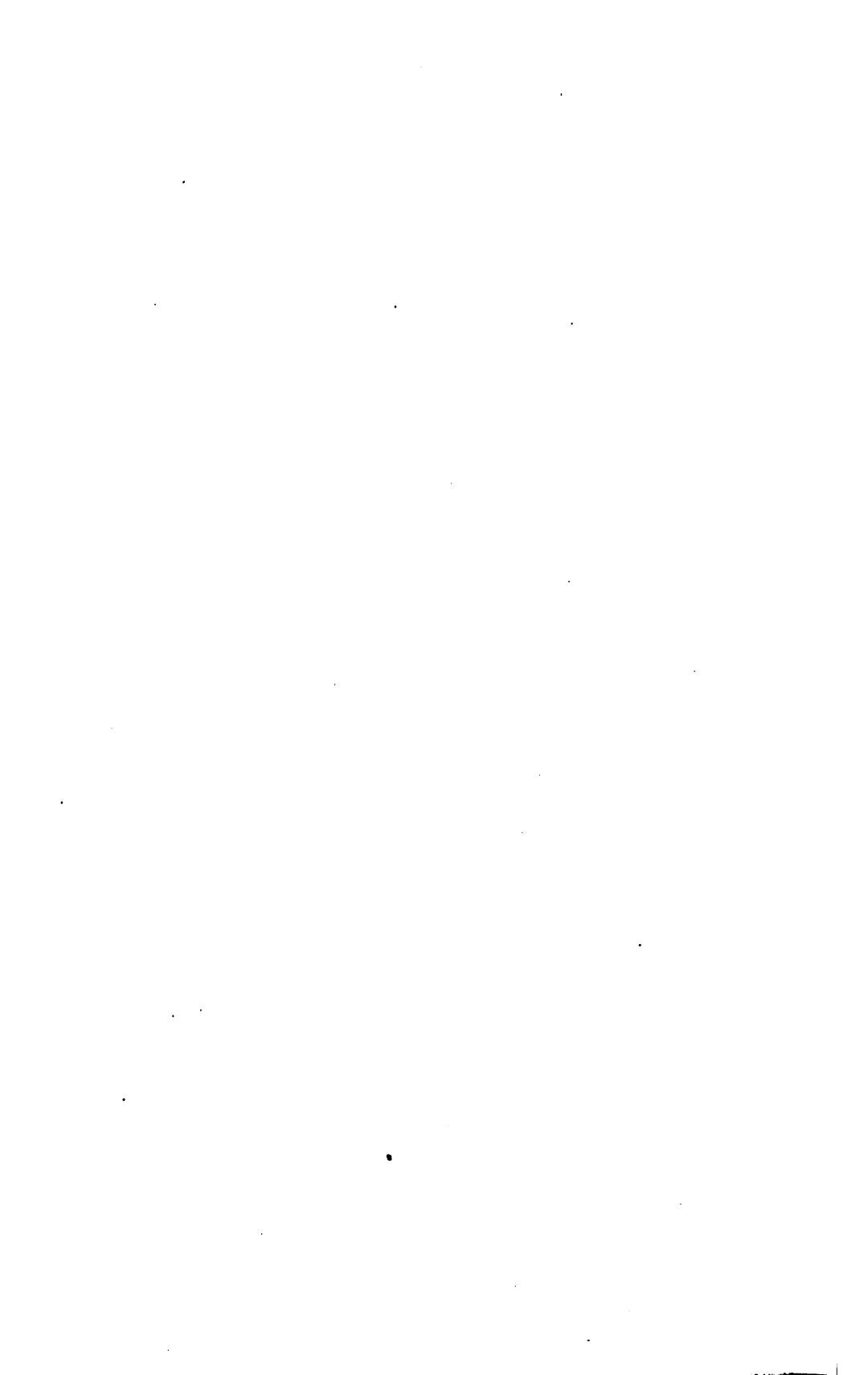


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A MANUAL
OF
TOPOGRAPHICAL DRAWING.

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PREFACE.

PROFESSOR SMITH's Manual on Topographical Drawing first appeared in 1854, and was at that time the only American treatise of any extent on the subject. In the course of time, the plates became somewhat worn with age, and required to be redrawn. The text, also, although retaining its original value as a presentation of the subject, was found to differ materially in respect to the order of presentation from that adopted in our leading technical schools. It was thought, therefore, that some advantage might be gained, and the work better fitted for use as a text-book, by recasting it in accordance with what experience had shown to be a more acceptable arrangement of its several parts.

The Editor, acting under the most liberal instructions from the Publishers, has endeavored to carry out the aims indicated above as thoroughly as time and other circumstances would permit, and with the loyal purpose of preserving intact as much of the Author's text as possible. While it is quite certain that there is still much room for improvement in the work as now presented, it is hoped that the changes and additions in both the text and plates will, in some degree, merit the approval of teachers.

The intermingling of two separate writings on almost every page renders it inexpedient to attempt to point out all the changes which have been made. Nor is any discrimination necessary; instructors and others who are familiar with the original work will readily perceive the alterations. It may be stated in this connection, however, that the extensive additions which have been made in the first part of the work have been introduced in the hope of rendering it more serviceable to those who may not have the benefit of a teacher's guidance.

The Editor desires to acknowledge his indebtedness to his

colleagues, Professor H. S. S. Smith, C.E., and Professor William Libbey, Sc.D. ; to the former, for the revision of Appendix B; to the latter, for aid in correcting proof and in other ways. Also to Professor J. E. Hilgard, Superintendent of the U. S. Coast Survey, and to his assistants, Messrs. C. O. Bontelle and E. Hergesheimer, for valuable information and suggestions. Plate VI., the Plate of Conventional Signs, and the Plates of Lettering are partial copies of Coast Survey engravings which were kindly furnished by the Superintendent. These copies must not be regarded, however, as samples of Coast Survey work; the process of reproduction which was employed failed, unfortunately, to reproduce the exact scales of shade and the delicacy of finish of the original engravings.

The cuts of Figures 1 to 14 in the text were kindly loaned by Messrs. Keuffel and Esser of New York City.

C. McM.

PRINCETON, N. J., *May* 4, 1885.

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TOPOGRAPHICAL DRAWING.

PART I.

DEFINITIONS ; MATERIALS ; INSTRUMENTS ; PRELIMINARY OPERATIONS.

CHAPTER I.

PRELIMINARY INFORMATION.

1. DEFINITION.—*Topographical Drawing*, commonly known as Map Drawing, is the art of representing on paper by lines, or some other conventional expressive means, the exact shape and figure of the earth's surface in a particular locality, as well as the dimensions and positions of all objects situated upon it.

2. OBJECTS TO BE REPRESENTED.—Two classes of objects present themselves for description ; the one *natural*, including, 1st.—mountains of every extent, their slopes, rocky sides, gorges and valleys, and in general, every inequality in the surface of the ground ; 2d.—bodies of water, as the sea, rivers, brooks, lakes, ponds, and marshes ; and 3d.—all natural productions or conditions of the ground, such as forests, heath, meadows, sand, etc. The other class comprises artificial works, such as buildings, inclosures, cultivation, roads, etc. Of this latter class, buildings may be distinguished according to the purpose for which they are used, as churches, mills, dwellings, etc. Inclosures may be variously represented, as ditches, hedges, walls or fences. The different kinds of cultivation are not usually discriminated, except when they are characteristic of the climate, or of the soil, in a particular locality, and when the scale of the map will allow them to be shown. Roads, or communications, are distinguished as turnpikes, railroads, canals, cross-roads, foot-paths, trails, fords.

3. OBJECTS REPRESENTED IN PLAN.—Every topographical draw-

ing addresses itself to the eye as if the spectator were situated above and looking down equally upon every part of it.* In representing, therefore, upon such a drawing the relative positions and the dimensions of objects, accurate measurements, according to some assumed scale, are used, and distances are laid off as in any other plan-drawing. But in expressing the nature of objects, many of them not being bounded by mathematical or regular lines, recourse is had to certain conventional means universally agreed upon among draftsmen. In some instances, the signs thus used are made to resemble, in some degree, the objects for which they stand, as in the case of forests, rocks, meadows, etc. In others, they are purely conventional, as in the case of hills, water, marsh, etc.

4. MODES OF EXECUTING DRAWINGS.—Topographical drawings are executed in pencil, crayon, pen and ink, or in water colors.

The pencil is used in field sketching and in the preliminary delineation of a drawing which is to be finished in ink or in colors.

Crayon is very rarely used, and then only in cases where a very large scale and other peculiarities of the drawing, together with considerations of economy, render a bold and inexpensive method of hill-shading admissible or desirable.

Mapping with pen and ink requires greater skill than any other mode of drawing, and admits of a beauty and delicacy of finish almost equalling that attained by the engraver's art. Its proper sphere is in the execution of finished drawings requiring great exactness of detail, and it is especially suitable for the preparation of maps which are designed for reproduction by engraving, or any of the lithographic processes.

Water colors are not as generally employed in topographical drawing as they might be, with advantage. They can be used

* There is a kind of perspective drawing known as a "bird's-eye view" that is occasionally met with. Such drawings, although they are capable of giving a general idea of the arrangement and kind of topographical features in any region, are nevertheless practically valueless for the purpose of exact measurement, owing to the variation of the scale in different parts of the picture. They are, moreover, open to the objection that, in a scene viewed in perspective, objects in the background are necessarily more or less obscured from vision by nearer or more conspicuous ones. They are at best merely sketches, and are usually excluded from the category of topographical drawings.

with great effect, and always with less labor and expense than the pen, in much of the mapping which is required in a civil engineer's practice. Besides the economy arising from their use, it may be stated in their favor that, if properly applied, with skilful shading and effective contrasts of light and shade, they will seldom fail to produce a pleasing and suggestive picture even to an unpractised eye. On the other hand, colored work is more suitable for use in connection with large scales than with the smaller ones, for which pen work is especially adapted. Moreover, the tints are somewhat apt to fade with age, though not to an extent calculated to mar the general effect; and the drawings, if soiled by handling, cannot be cleaned without material injury as readily as pen drawings.

In Landscape Architecture, as it is styled in this country, a knowledge of colored topographical drawing is of the highest importance.

CHAPTER II.

DRAWING MATERIALS AND INSTRUMENTS.

5. **DRAWING BOARDS** are of various sizes. They are usually made of thoroughly seasoned white pine, and should be free from knots and pitchy spots. In order to prevent their warping, cleats, crossing the grain, are either screwed or glued to the backs, or are fastened with tongue and groove joints to the two opposite edges. The tendency to warp is diminished by making the board out of narrow strips instead of wide ones. For very large drawings, drawing boards should be exceptionally heavy and unyielding.

6. **PAPER.**—There are several kinds of paper which are suitable for mapping. The essential quality of such paper is a hard surface of uniform texture on which ink or water colors will flow smoothly ; which will neither be easily roughened by the scratching or cutting action of the pen, nor absorb tints, that is, allow them to strike into the body of the paper ; and which will bear any ordinary application of the rubber without sensible injury.

7. The paper most commonly used is *Whatman's*, either *hot* or *cold pressed*. The former has a smooth surface and is especially adapted for fine pen work ; more particularly for engravers' and lithographers' copies or models. The latter is of a very dense texture, possesses a hard and coarse-grained surface, and is especially designed for water-color drawings ; although, owing to its superior qualities, it is frequently used by draftsmen for both purposes. Whatman's paper bears the impression, "WHATMAN," or "WHATMAN TURKEY MILLS," which can be read by holding the paper between the light and the eye.

The following are the names and dimensions of the sheets :—

Cap	13x17 inches	Super Royal.....	19x27 inches
Demy.....	15x20 "	Imperial.....	22x30 "
Medium.....	17x22 "	Atlas.....	26x34 "
Royal.....	19x24 "	Double Elephant.....	27x40 "
Antiquarian.....		31x53 inches.	

The thickness of the paper increases with the other dimensions, and, usually, also the coarseness of the grain. For water-color drawings, therefore, both thickness and coarseness of surface being desirable, it is recommended that whenever the space to be occupied by the drawing calls for the use of the smaller sheets, the larger sizes, halved or quartered, as each case may require, be employed instead. Thus, a sheet of Super Royal will make a little more than two sheets of Cap; Atlas, four sheets of Cap, etc. This course is more expensive, but frequently more satisfactory, than that which would naturally be pursued.

8. *Antique* or *Egg-shell* paper—a favorite among landscape painters—may be used with considerable advantage in mapping with colors, especially in designs for the ornamental arrangement of grounds. The inequalities of the surface give, on the one hand, a peculiar softness and beauty to a picture, and, on the other, aid considerably in producing bold or striking effects. The paper is not suitable for drawings executed to small scales.

9. There are other kinds of paper which are sometimes used for topographical drawings. Among these the so-called "*German*" or "*Imported Paper*" is best known. It is lacking, however, in hardness of surface, and in moist weather is apt to prove troublesome by clogging the pen with its fibres, which are easily detached in such weather.

10. PREPARING THE PAPER.—Drawing paper may be used plain, that is, as it comes from the stationer, in which condition it is fit only for pen work; or it may be stretched on a drawing board, or mounted on muslin, when it becomes suitable for either pen work or coloring.

11. When used *plain*, it is best to fasten down the corners and, if desirable, intermediate points along the edges, to a drawing board by means of thumb tacks. The paper will thus be saved from becoming wrinkled or broken by use, and if the board be of suitable size, the ease of turning the drawing and moving it about will be considerably increased. The side of the paper on which the water-mark can be read should, in all cases, be placed uppermost.

12. Drawing paper can be *stretched on a board* by first sponging the paper lightly with clean water—if hot pressed, on the under side, if cold pressed, on the upper surface—and, after it has been expanded by the moisture and the wrinkles caused by

the wetting are disappearing, fastening the edges down with glue or tolerably thick mucilage.

In the case of the Whatman papers this process is facilitated by trimming off their hard, irregular edges to a depth of from one-eighth to one-quarter of an inch, before sponging the paper. The width of the gummed strip on the edges need not be greater than three-quarters of an inch, but it should be well defined by folding the edges over. The gum or paste is then applied to them, and they are merely turned down and smoothed with a paper cutter, or some similar article; care being taken to interpose between it and the moist edges a clean piece of blotting paper, and to make all rubbing movements of the hand over the blotter outward, so as to press out all surplus mucilage from under the sheet. The paper, while wet, should not be stretched by hand; it will expand quite enough by the action of moisture.

After glueing down the edges, any little pools of water remaining on the surface should be soaked up with the sponge, and the paper left in a horizontal position to dry slowly.

13. Another mode of stretching paper consists of thoroughly wetting all but the folded edges, which are then gummed and pressed down on the board (drawing them lightly outward) and smoothed with a paper cutter, or the smooth handle of a knife or eraser, until they adhere throughout. Inasmuch as, by this process, the edges will be drier than the body of the sheet and, therefore, will have expanded less, the strain on the sheet when dry will be slighter than if it had been stretched by the first method, and the consequent shrinkage of the finished plate when cut off from the board will be less. This is an important consideration when it is desirable to express the scale of the drawing in the form of a fraction, or as a certain number of feet, yards, metres, or miles, to the inch, for the shrinkage of the paper may make an appreciable alteration in the scale of the finished drawing.

14. *Mounted Paper.*—The heavier kinds of paper adapted to topographical drawing, excepting Whatman's hot pressed, can be procured at the present time *with muslin backing*, if so desired. Yet, as it is sometimes more convenient to *back* or *mount* the paper only as occasion requires, it is desirable that every draftsman should be acquainted with the proper mode of doing this promptly and neatly. The muslin (white, and of a coarse quality) is cut

not less than four inches wider in every direction than the paper which is to be mounted on it. It is laid flat on a clean board or table, and the edges to the depth of about an inch are smeared with binder's paste and pasted down, straining the cloth slightly so that it will lie smooth. A row of fine tacks may be added along each edge for security. After sponging the cloth lightly with a damp sponge (rubbing always in the same direction), until the fibres have all been laid flat and all wrinkles and folds have disappeared, the muslin is ready to receive the paper.

The latter, having been spread face downward on a clean table, is thoroughly painted on the back with binder's paste (free from lumps) applied with a large flat brush—the painting being continued until all wrinkles have disappeared. The paper is then laid carefully, face uppermost, in the position it is to occupy, and one-half of the sheet, being lifted with one hand, is gently smoothed and pressed down on the muslin with the other, beginning at the middle and proceeding outward toward the edges. The effect of this is to press out air bubbles, which, otherwise, would be formed between the paper and backing and would prevent a thorough adhesion of the two. The other half of the sheet is then treated similarly, and it is left to dry. A good substitute for the hand in smoothing the paper is a clean and dry flat brush about seven or eight inches in width.

Paper thus treated is much more durable than other kinds, and is especially fitted for drawings which have to be very frequently consulted, or which are liable to receive rough treatment. It may be left on the board until the drawing is finished, or it may be detached immediately after drying; but in the latter case the backing should be of the heaviest grade of muslin, otherwise the application of colors will be apt to distort the sheet permanently.

15. PENCILS.—Only the best and hardest kinds should be used. For topographical drawing, they should be kept carefully sharpened to a point by rubbing them on a piece of fine emery paper, and should be applied to the drawing with very light pressure—only sufficient to render the lines visible. It should be borne in mind that careful pencilling is an indispensable preliminary to fine drawing, and pencil lines should therefore be drawn with great exactness, to serve as guides for the pen, and of the requisite lightness which will avoid making dents in the paper and enable

the pencillings to be erased with the least possible injury to the pen work which succeeds them. Ruled pencil lines should be drawn a little beyond their exact length, for in going over them afterwards with a pen, their intersections can be more readily distinguished by means of these projecting ends of the lines.

16. RUBBER should be of the finest grain and absolutely free from grit. The so-called "velvet rubber" is probably the best for fine drawings.

17. PENS.—The best are *Gillott's Lithographic pens*. Next in grade to these is the *mapping pen*, and lastly the *crow quill*, all by the same maker. They are all suitable for fine work, yet the first kind is decidedly superior to the others, and is recommended, especially to beginners, as being particularly valuable in training the hand to execute delicate work. The mapping pen may also be used for this purpose, but not the crow quill. Its greater rigidity, by calling for greater effort in making heavy lines, is apt to lead the pupil into either a coarser or more spiritless style of drawing than the more flexible pens.

The crow-quill pens, however, are excellent pens for lettering, inasmuch as they admit of being used with up strokes, while the others do not. For drawings made on a large scale, Gillott's writing pen, No. 303, is occasionally used with very good effect.

18. In topographical drawing, all ink lines drawn by hand, that is, without ruling, have to be made slowly and steadily, and are, therefore, more easily drawn *towards the body*; and with this view, the paper should be turned on the table in any way that will facilitate that mode of drawing.

19. INDIA INK is sold in oblong cakes of various sizes and qualities, ornamented with Chinese characters. It should be of the best quality, which insures its quick and perfect mixture with water, and, for use, should be rubbed up *perfectly black* with four or five drops of water, in an ink slab or saucer. It is important to do this properly, inasmuch as pale ink makes the boldest drawings look weak; while, on the other hand, if the ink is too thick it will be constantly clogging the pen by drying. To test its blackness when mixed, take some in a pen, make a pretty broad mark with it upon white paper, and wait until it dries, when it will display its true strength. A little experience will render the test unnecessary.

20. *Liquid ink* can be prepared by boiling fragments of a

cake in about four times their bulk of pure water until they are completely dissolved, and the mixture is of the right consistency. If the ink is of good quality the operation occupies but a short time, and there is little uncertainty as to its results. The prepared ink can be kept in a corked vial, whence a few drops may then be poured out into the slab, as required. After a week or ten days, liquid ink, thus prepared, will acquire an offensive odor. This source of annoyance may be partially avoided by the addition of a drop or two of carbolic acid to the contents of the vial; but should it recur, the only remedy is to boil the ink again, after adding a little clean water. A test tube, such as is used by chemists, is the most convenient vessel in which to boil the ink.

A very fair quality of liquid ink may be obtained from dealers in artists' materials.

21. RULERS.—For small drawings, a pair of triangles of pear-wood, or German silver (*Fig. 1*), together with three or four irregular curves (*Fig. 2*), will be quite sufficient. In choosing the latter, some reference should be had to the nature of the ground which is to be represented, and to the scale of the map. Generally, it will be safe to select two with very sharp curves, and the others of more rounded outline. For larger drawings, a T-square (*Fig. 3*) or a straight-edge, the latter preferably of steel, must be added. It is also often convenient to have two pairs of triangles, one large

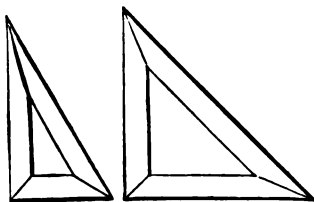


FIG. 1.



FIG. 2.

and the other small; and sometimes, a parallel ruler (*Fig. 4*), if in good order, is found to be useful; but parallel rulers having any play or lost motion in the joints should be discarded. The edges of the T-square, straight-edge, parallel ruler and triangles should be perfectly

straight and smooth, and the angles of the latter true; the irregular curves, also, should be without breaks or angles on the edges, and should have a smooth finish.

22. *The modes of using triangles* for drawing parallels and perpendiculars need a short explanation. In order to draw a parallel to a given line, place an edge of one

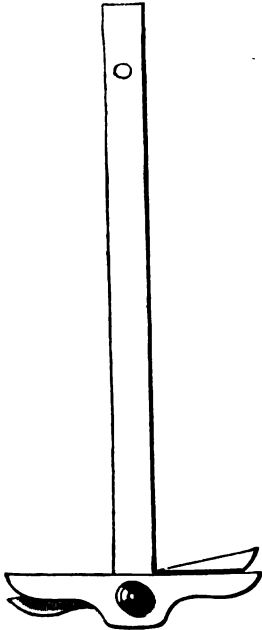


FIG. 3.

of the triangles coincident with the given line. Holding it firmly in this position, bring to bear against one of its other edges an edge of the other triangle, as a rest or guide. If the second triangle is now held firmly, and the first is slid along its guiding edge, the edge which coincided with the given line will move parallel to that line, and will give the directions of the line required to be drawn.

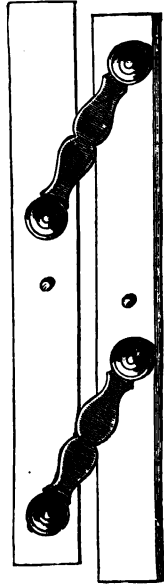


FIG. 4.

The simplest way of drawing a perpendicular to a given line is to place one of the adjacent

edges of the right angle of triangle No. 1 against the given line, as in drawing parallels; then bringing up No. 2, as before, for a guide, slide No. 1 upwards, when its other side about the right angle will give the direction of the perpendicular. Still another way is to place the hypotenuse (the longest side) of triangle No. 1 against the given line, and to bring up No. 2 against the longer edge about the right angle of No. 1. Now, turn the latter about its right angle until its shortest edge bears against the guiding edge of No. 2. The hypotenuse of No. 1 will, in this position, be perpendicular to the given line, and by sliding the triangle back and forth the required line may be drawn.

The modes of using T-squares, and triangles with T-squares and straight-edges, will readily suggest themselves to the pupil.

23. INSTRUMENTS.—But few of the so-called mathematical instruments are needed by a topographical draftsman, but those used should be of good quality and kept in perfect condition.

Those commonly needed are, a small *right-line pen* (*Fig. 5*) for fine lines in the interior of the map ; a larger one for borders and heavy lines of considerable length ; a pair of “*compasses*” or “*dividers*,” either plain or with hair spring, for spacing off distances (*Fig. 6*) ; a large pair of dividers with pencil, pen, and needle-points, for drawing circular arcs ; a *road pen*, consisting

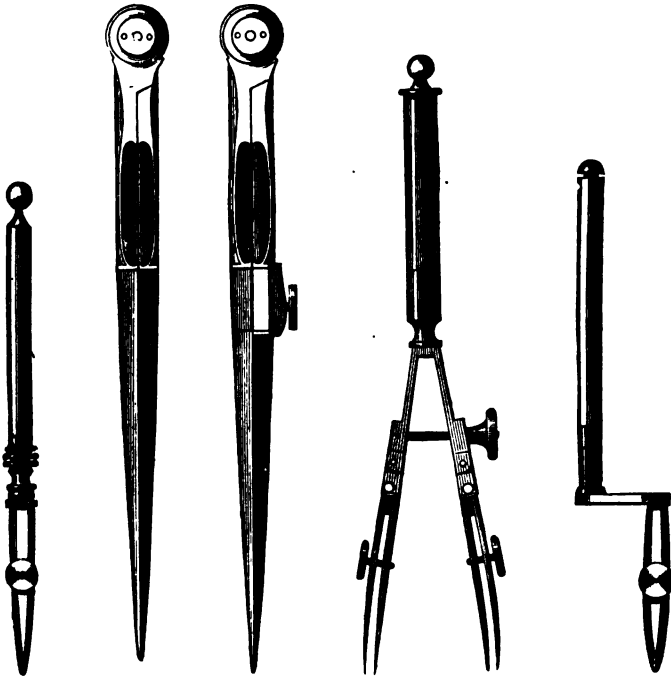


FIG. 5.

FIG. 6.

FIG. 7.

FIG. 8.

essentially of two small right-line pens fixed to the same handle, the space between them being adjusted to correspond to the width of the road by a screw, whereby they may be drawn nearer, or moved farther apart (*Fig. 7*) ; and, finally, a *curve pen* or *contour pen* for drawing curved lines (*Fig. 8*). Those more rarely used are, a *three-legged compass* for copying drawings (*Fig. 12*) ; and, for reducing maps, *proportional compasses* (*Fig. 13*), or a *pantograph* of fine quality (*Fig. 14*).

24. All ruling pens should be frequently sharpened by gently passing the outer faces of the blades (the pen being slightly open) with a rocking or rolling motion of the fingers or hand, over a

very fine-grained oil-stone, used dry. When properly ground, the points will be tapering, *of even length*, and sharp enough to make, with a light pressure, a fine cut on the finger-nail. Another way of testing their sharpness is to view them endwise in a strong light; well-sharpened points will reflect no minute dots or lines of light from the extremities, while with blunt pens the reflections will be plainly visible. The final test, in every case, is to charge the pen with ink and to draw a series of lines, beginning with moderately broad ones and gradually reducing their thickness down to the finest. If the lines are all sharp, clear, and unbroken, the pen is in the proper condition.

25. In ruling with the pen both points must press equally on the paper. A properly ground pen should, therefore, be moved in a plane which is perpendicular to the paper; but it may be held more or less slanting in the direction of the line which it is tracing, according to the convenience of the draftsman. The movement of the pen should be along that edge of the rule which is furthest away from the body, and from left to right, except in the case of left-handed persons, when the direction will be reversed. The drawing should be turned freely to admit of greater ease in ruling.

26. WATER COLORS.—These are either *hard* or *moist colors*. The former are sold in half or whole cakes; the latter, in half or whole pans. The moist colors are becoming very popular, and are generally preferred by draftsmen owing to the greater ease of mixing them. The most convenient way of keeping them is in a japanned tin box designed for the purpose. If cakes are used, they should be enveloped in light wrappings of tinfoil, applied with a little mucilage, to prevent them from crumbling.

The following water colors are needed: *Hooker's Green No. 2, Indigo, Prussian Blue, Carmine, Gamboge, Yellow Ochre, Burnt Sienna, and Sepia*. It is sometimes convenient, also, to be provided with *Cobalt Blue* and *Burnt Umber*. None but the best quality of colors should be used.

The use and significance of these colors will be described in Part II, Chap. IV.

27. BRUSHES.—As regards material, two kinds of brushes are used, viz., *camel's hair* and *sable*. The latter is the stiffer of the two, has a fine point, and is employed in touching up and finishing details. The former has a blunter point, has finer and

softer hair, and is especially suited for laying on, or sweeping, tints over broad surfaces.

As to size, these brushes vary greatly. The only advice which can be given on this point is of a general nature, viz., that the size of the brush, more particularly in tinting, should be somewhat in proportion to the surface to be covered. Large brushes are unwieldy for small surfaces, while small brushes used on large surfaces require to be replenished too frequently before the tint is finished, causing it to be uneven. Before purchasing brushes, they should be tested as to the condition of their points, by dipping them in a tumbler of clean water and smoothing them gently on the edges of the glass. If, under this treatment, the brushes are drawn to a smooth point, they are in good condition ; if not, they should be rejected.

In addition to the preceding, a flat brush, from one to two inches wide, will be necessary for moistening large surfaces as a preliminary to tinting.

28. The following are some minor articles which it is often convenient to have at hand :—an *erasing knife*, for erasing ink lines ; a *horn centre* which, placed over the centre of an arc, will preserve the paper from being injured by the needle-point of the compasses ; three or four *lead weights* ; a cake of *mouth-glue*, for attaching tracing paper to drawings ; and a little prepared *ox-gall* to mix with the ink or colors in case the paper should be greasy.

CHAPTER III.

SCALES ; THEIR CONSTRUCTION AND USE.

29. Before projecting upon a map the data collected by a survey, it is necessary to decide upon some scale of horizontal distances, suited alike to the purposes for which the map is intended, and to the nature and amount of detail that it is required to represent. If the scale is too small, it banishes many details that might be desirable ; if too large, it produces an unwieldy drawing. In order that a scale may be a convenient one for use, it is necessary, on the one hand, that the dimensions measured on the ground should be converted, without calculation, or by an easy effort of the mind, into the corresponding dimensions on the map ; and on the other hand, that the dimensions of the ground should be just as easily inferred from those of the map.

For example, in the scale of one foot to a thousand feet, or $\frac{1}{1000}$, one hundred feet of the ground is represented by one-tenth of a foot on the map, or .1 ; one hundred and fifty feet, by .15 ; and one hundred and seventy-eight feet, by .178, or 178 thousandths of a foot, which bears so close a relationship in its figures, to the distance measured on the ground, that it can at once be taken in the dividers from a scale having decimal divisions of the foot. On the contrary, a scale is inconvenient for use, when its ratio is such a number as $\frac{1}{7920}$, or eight inches to a mile, or $\frac{1}{10560}$, which is six inches to one mile. The long measure used in our country, viz., miles, yards, feet, and inches, has been the means of retaining these arbitrary ratios in use among our draftsmen ; but in the service of the United States Coast Survey, the decimal scale is adopted for all maps. The French long measure, being expressed entirely in decimals, makes the application of the decimal scale perfectly appropriate ; but with us, common usage renders the employment of the British units of length a necessity at present.*

*The American Society of Civil Engineers has recommended that every drawing should have two scales represented on it ; the one, corresponding to the metric system, the other, to the common (British) system of linear measures.

30. Draftsmen's Linear Measures—technically known as *scales*—marked with scales of different denominations are manufactured in great variety. They are made of ivory, box-wood, steel, brass (nickel plated), hard rubber, or paper. As to form, they

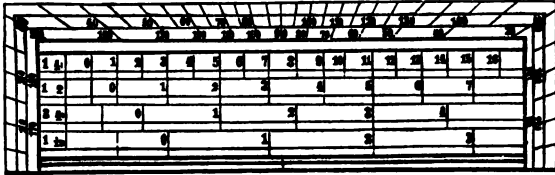


FIG. 9.



FIG. 10.

are either *flat scales*, i. e., rulers graduated on the faces or on the edges; or they are of the kind known as *rectangular protractors* (Fig. 9), with linear scales on the faces and angular graduations on the edges; or they are of triangular cross section, and are graduated on the edges (Fig. 10).

31. These measures are applicable to the construction of the pencilled outlines of small maps, and even of large ones, whenever extreme accuracy is not required. The student will understand this qualification better when it is stated that, however accurate may be the graduations of the scale which he is about to employ, discrepancies in the lengths of the lines on the drawing will be likely to arise from the expansion and contraction of the paper by variations in the humidity of the atmosphere. In large drawings, this is especially noticeable, and is a source of either inaccuracy or delay. Therefore, where great exactness is needed, it is best to construct a scale of distances, for temporary use, on the same paper with the drawing. This is called the *scale of construction*. Another, less minute in its subdivisions than the preceding, called the *scale of distances*, is afterwards drawn in a suitable place within the borders of the finished map, and is used only in finding and comparing distances on the paper.

32. *To make the scale of construction (Pl. VIII. Fig. 47).*
 Draw a right line with the pencil and, supposing, for example, the scale to be $\frac{1}{1000}$, divide it into equal parts, each one-tenth of a foot in length. This ought to be done with the graduated edge of a good scale. Above these points of division write the numbers 0, 100, 200, 300, &c. Prolong the measured line on the left of the zero, and lay off the distance of one tenth of a foot, which must be divided into ten equal parts, numbered in regular order from the zero to the left. Let fall perpendiculars from every point of division, then draw ten other lines below, parallel to the first, and equidistant from each other. This equal spacing may be two tenths of an inch, or less, or more, provided it be constant for each space. Now in the space of the $\frac{1}{10}$ of a foot which lies on the left of the zero, draw the diagonal lines, as in the fig., by joining the first division on the left of zero (the point *c*) with the point *b*, and drawing through the points 20, 30, 40, &c., lines parallel to *b c*. From this construction, and the properties of similar triangles, it is evident that the part of the line AB (the second line from the bottom), which is included between the sides of the triangle *o b c*, is equal to one-tenth of *o c*, the base of the triangle; that the corresponding part of the third line CD, is equal to two-tenths of the base, *o c*, of the fourth line, it is equal to three-tenths, and so on. But the base, *o c*, of the triangle is the $\frac{1}{10}$ part of a foot, therefore the parts of the horizontal lines intercepted by its sides are, respectively, $\frac{1}{1000}$, $\frac{2}{1000}$, $\frac{3}{1000}$, &c., of a foot, which fact is expressed by the figures 1, 2, 3, 4, &c., upon those lines. If it be required to find from this scale, a distance of 277 feet, place one foot of the dividers at *d*, on the vertical line numbered 200, and upon the horizontal line numbered 7, then place the other foot at *e*, at the intersection of the same horizontal line with the *diagonal* numbered 70, and the distance will correspond to 277 feet. This is called the *diagonal scale of equal parts*, and a scale thus constructed is applicable to all decimal ratios, the numbering only changing with the ratios.

33. If the distances are expressed in other terms than in feet, the top line of the scale must be divided according to those terms. For example, if the scale were one inch to a hundred feet, the upper line of the scale must be divided into inches, and the inch on the left of the zero into tenths. Then the ten hori-

horizontal lines, and the diagonals, will express hundredths of an inch, or one foot on the ground.

34. Other ratios may be expressed on the diagonal scale by the following method : Suppose it is required to construct a scale of 24 inches to 1 mile, which shall be capable of measuring any odd number of feet. The ratio of this scale is $\frac{1}{2880}$, that is, one foot of the drawing corresponds to 2880 feet on the ground. Then one inch of the drawing (and of the scale to be constructed) corresponds to 240 , or 220 feet, and one-tenth of an inch to 22 feet. Draw the line of the scale as before (*Pl. VIII. Fig. 48*), divide it into inches, and number the points of division from the zero to the right, 0, 220, 440, 660, 880, &c. Divide the inch on the left of the zero into ten equal parts, and number them 0, 22, 44, 66, 88, 110, &c., from the zero towards the left. Now draw *eleven* parallel and equal-spaced lines below the first one, and draw the diagonals in the space on the left of the zero. It is evident, from an inspection of this scale, that the parts successively cut off upon the horizontal lines by the sides of the triangle *obc* are, commencing from the line next to the lowest, respectively equal to $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, &c., of 22 feet, or the base of the triangle, which is one-tenth of an inch ; or, in other words, they are respectively equal to 2, 4, 6, 8, 10, &c., feet. Single feet are found by placing the compasses midway between the horizontal lines. This scale is very much inferior, in point of convenience, to the decimal scale, on account of the complicated numbers which express its divisions, the addition and subtraction of which cannot be readily effected by a mere mental operation.

35. Other scales may be constructed as follows:—12 inches to 1 mile, or $\frac{1}{2880}$: Divide the line into inches, and number the divisions 0, 440, 880, &c. Divide the inch on the left of the zero into *eight* equal parts, each of which will be 55 feet ; draw *eleven* lines below and parallel to the first line, and draw the diagonals. The parts cut off on the horizontals by the sides of the triangle, will be successively 5, 10, 15, 20, &c., feet. 8 inches to 1 mile, or $\frac{1}{3600}$: Divide the scale into inches, and the left hand inch into *ten* parts. Draw *eleven* horizontal lines below, and the diagonals ; the parts then cut off will be 6, 12, 18, 24, &c., feet successively. 6 inches to 1 mile, or $\frac{1}{4800}$: Divide the line into inches, and the left hand inch into *eight* equal parts. Draw *eleven* horizontals. The distances cut off will be 10, 20, 30, 40,

&c., feet. 4 inches to 1 mile, or $\frac{1}{15840}$: Divide the line into inches, and the left hand inch into eight parts. Draw eleven horizontals, and the distances will be successively 15, 30, 45, &c., feet. The subdividing qualities of all these scales may be increased by augmenting the number of the horizontal lines. For example, in the scale of 6 inches to 1 mile, if there were 22 horizontals instead of 11, the successive distances would be 5, 10, 15, 20, &c. If the horizontals were doubled in number in that of 8 inches to a mile, the distances would be 3, 6, 9, 12, &c., and if they were trebled in the scale of 4 inches to a mile, or made 33 in number, we should have distances of 5, 10, 15, 20, &c., feet.

36. The reason for drawing eleven horizontals in some of the above described scales is, that the number 11 is an exact divisor of the number of feet represented by one of the divisions of that part of the line which is on the left of zero. To take one more example. 5 inches to 1 mile, or $\frac{1}{12176}$: Here one inch of the scale corresponds to 1056 feet, or $\frac{1}{8}$ of an inch to $\frac{1056}{8} = 132$ feet. We divide the inch on the left of the zero into eight parts, and for the number of horizontals we seek the largest desirable factor of 132, which may be either 22 or 12. If we draw 22 horizontals, the distances given by the diagonals will be 6, 12, 18, 24, &c.; but if 12 horizontals are used, they will give distances of 11, 22, 33, 44, &c., feet. It is evident, also, that the inch on the left of zero must be so divided, that the number of its equal parts will be an exact divisor of the number of feet corresponding to the inch, as above. 1056 is a multiple of 8, and not of 10, hence we divide the inch into eighths, each of which is 132 feet.

Unless these divisors can be found, the scale cannot be made to express whole numbers by its smaller divisions, which is a great defect. It is to be hoped that the decimal scale and ratio will eventually be adopted.

37. In adapting the scale to the uses of a map the following general proportions may be observed: A map constructed on a scale of half an inch to a mile, or $\frac{1}{15840}$, will admit the representation of all towns, villages, main roads, the principal cross-roads, and every considerable mountain and stream. On a scale of one inch to a mile, or $\frac{1}{7920}$, besides these, farms, woods, isolated buildings, every stream of 600 feet in length, and every

hill of 100 feet in height, can be represented. On a scale of two inches to a mile, or $\frac{1}{31680}$, the various features of the ground can be clearly and accurately presented, also every stream of not less than 300 feet in length, every pond of more than 50 feet broad, besides all roads, isolated buildings, &c. The scale of six inches to a mile, or $\frac{1}{10560}$, is well suited for the complete delineation of a country. Scales for projecting experimental surveys for civil purposes very seldom exceed twelve inches to a mile, or $\frac{1}{5280}$. Larger scales than these are only used in proportion to the amount of detail required. The decimal scales corresponding nearly to these are $\frac{1}{100000}$, $\frac{1}{10000}$, $\frac{1}{30000}$, $\frac{1}{10000}$, and $\frac{1}{5000}$. The smallest publication scale of the U. S. Coast Survey is $\frac{1}{80000}$, which is also the scale of the new map of France.*

38. To construct a *scale of distances* (Pl. VIII. Fig. 46) draw a horizontal line, and subdivide and number it as in the previous case. Then ink it with the right-line pen, and draw beneath it a heavy line, say, two-hundredths of an inch wide, and at a distance below the first line equal to its width. This heavy line may, or may not, be carried to the left of the zero of the scale, according to the taste of the draftsman. The former course is sometimes preferred, as it admits of applying a fine arrow head, or some other form of ornamental finial, to the ends of the scale. Then with a right-line pen, rule the divisions before measured off, drawing them perpendicular to, and across, both lines of the scale. If it should be required, for example, to take from this scale a distance of 180 feet, place one foot of the dividers upon the point marked 100, and carry the other beyond the zero to the left, until it comes to the division marked 80. The compasses will then include the required distance. Any odd number of feet will be found by supposing each of the small spaces on the left of the zero to be divided into ten equal parts, and placing the foot of the compasses accordingly. The numbers and the lettering, as indicated, should then be inked in very carefully.

39. *Angular scales* or *protractors* (Figs. 9, 11) differ greatly in quality and cost, some of them being very expensive and

*The reader is referred to the beautiful maps of the United States Coast Survey, and to the very elegant detailed map of France, both in course of publication, as admirable illustrations of topography.

provided with attachments and refinements which are altogether out of proportion to the thickness and natural inequalities of the finest pencil or ink lines. The protractors which are most generally used are also the cheapest. They are full circles of

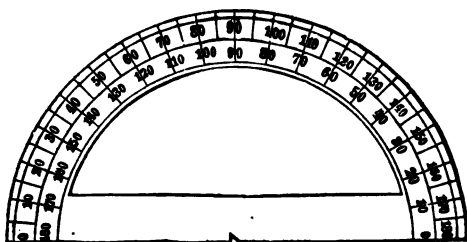


FIG. 11.

paper from six to fourteen inches in diameter graduated to half or quarter degrees according to their size. The materials are drawing paper, bristol board, or tracing paper. The first two

should have small lozenge-shaped openings at the centre, with sides about one-tenth of an inch long, in order to display the vertex of the angle. The transparent protractors on tracing paper admit of great accuracy, and are very highly prized.

40. TO LAY OFF AN ANGLE ;—*with a Protractor.* Place the protractor over the given line from which the angle is to be laid off, so that the 0° and the 180° points cover the line (produced if necessary) while the centre of the protractor is directly over the given point. Prick off on the paper, with a fine needle point, the required angle, and draw a fine line through the point thus marked and the given point. If the protractor is a full circle, it is more correct to prick off, also, a second point directly opposite, or 180° from, the first. The right line connecting them should pass through the given point, and being longer, will give the direction of the required line more exactly.

41. It is sometimes more convenient, in using a paper protractor, to fasten it in some suitable place near the border of the map, the diameter passing through the 0° and 180° marks being strictly parallel to the meridian of the map. A line making any given angle with the meridian can then be drawn through a given point, by applying the edge of a triangle, or parallel ruler, to the points on the protractor corresponding to the given angle, and then shifting the edge (as described in drawing parallels) until it passes through the given point, when a line drawn along the edge will have the required direction.

42. *By Means of a Scale of Chords.*—This scale is to be found, on the larger sizes of ivory protractors on the line

marked C H O. Take with the compasses (furnished for the purpose with needle and pencil points) the distance from 0 to 60, from the above scale and, applying the needle point to the vertex of the required angle, sweep, with the above distance as a radius, an arc of indefinite length cutting the line from which the angle is to be laid off. Then, applying the compasses to the scale, take therefrom the distance from the zero to the number corresponding to the angle which is to be laid off. With this as a new radius, and with the centre at the intersection of the first arc and the given line, sweep a short arc cutting the first. A straight line passing through the intersection of the two arcs and the first centre (the vertex) will make the required angle with the given line.

43. *By Latitudes and Departures.*—When the Latitudes and Departures of the lines of a survey have been calculated, they afford the easiest and most accurate mode of laying off, not only the angles, but also the lengths of the lines or courses. A description of the best method is given in Chapter IV., *Par.* 64. There are other methods of laying off angles, but they are seldom used.

CHAPTER IV.

COPYING, REDUCING, AND ENLARGING DRAWINGS, AND PLATTING.

44. TO ADJUST THE MARGINS.—Find the intersection of the two diagonals of the paper by laying a rule from corner to corner and drawing light pencil lines near the middle. Their intersection will be the middle of the sheet, with which the centre of the finished plate ought, usually, to coincide. Draw through this central point a line parallel to the lower edge of the sheet, then perpendicular to this, and through the same central point, another line. Lay off, from the central point, to the right and left, on the horizontal central line, distances equal to half the length of the rectangle intended to contain the drawing, and through the points thus found, draw lines parallel to the upright central line; these will be the indefinite upright sides. Then through two points on the upright central line, at distances above and below the centre equal to half the altitude of the required rectangle, draw lines parallel to the horizontal central line; they will complete the rectangle.

If the paper is stretched on a board, a rectangle should first be drawn in pencil, the sides of which are just within the gummed edges, and the diagonals of this rectangle should be used instead of those of the sheet.

45. COPYING DRAWINGS.—*With the aid of Squares.* The ease and accuracy of copying the outlines of a drawing correctly may be greatly increased by the use of some simple arrangement of auxiliary lines which are drawn, in duplicate, both on the model and on the plate on which the copy is to be made. The most convenient method is to divide the drawing into squares whose sides are, in direction, parallel to the margin lines respectively, and, in length, some fraction of the shorter side of the drawing, such as $\frac{1}{4}$, $\frac{1}{10}$, or $\frac{1}{16}$. It is evident, however, that any kind or number of lines drawn upon the model will answer the

same purpose, provided the proposed copy is treated in exactly the same manner. Having, then, these similar systems of lines, it will be easy to cause the outlines in the copy to pass through points in the squares corresponding to those of the model. This process at once suggests the method of enlarging or reducing a drawing by increasing or diminishing the sides of the corresponding squares. In comparing the proportions of similar drawings, linear measure is always used; e. g., a drawing is said to be twice the size of another when it is twice as high and twice as wide, though it contains four times the surface. The squares must be drawn in pencil, and lightly, as they would disfigure the drawing if they could not be entirely removed.

46. In copying lines which are so close together that many of them are contained in a single square (sometimes the case with horizontal curves), the square can be subdivided on the model, and on the copy, by joining the middle points of the opposite sides, so that one square will be made into four. Or, only alternate curves may be studied and drawn, and the intermediate ones can, afterwards, be easily introduced.

47. If the three-legged compass is used, place two of the legs on the corners of the square on the model, and the third on the point which is to be transferred to the drawing. Then, carefully preserving the relative positions of the legs, apply them to the corresponding points on the copy, when the third leg will indicate the position of the required point.

48. The preceding methods consist, essentially, of platting a series of points which are afterwards connected by lines, either straight or curved the latter being drawn by the aid of suitably curved rulers, or simply by the aid of the eye. In the latter case, both the model and copy must be so arranged before the draftsman that the line he is engaged in drawing may be parallel to the one he is copying.

The draftsman should begin with the principal lines of the drawing; for example, if a broad stream or an extended sheet of



FIG. 12.

water be represented, begin with that; then proceed to the roads, and smaller streams. Prepare everything completely in pencil before taking up the pen to finish, for in doing so the progress of the work is more satisfactory and apparent.

No line that is meant to be a straight line should be drawn *with the free hand*. Right lines, whether in ink or pencil, must invariably be ruled, no matter how short they may be; and if in ink, should be drawn with the right-line pen. Nor should a right angle ever be guessed at. All square corners, whatever be the shortness of the lines forming them, must be constructed with the proper instrument. Parallel lines, also, no matter how short, must be constructed.

Let the drawing be kept clean. All parts of it, except that which occupies the immediate attention of the draftsman, should be well covered with clean white paper; and a piece of thin paper should be constantly interposed between the drawing and the draftsman's hands.

49. *With the aid of Tracing Paper.*—In employing this method, either separately or in connection with that just described, spread the tracing paper perfectly smooth over the model and glue down the corners very lightly with mouth glue, or hold it firmly in place with leaden weights; then, with a sharp, hard pencil, or, still better, with pen and ink, go over the lines which are to be transferred. Removing the tracing from the model, spread it, face downward, on a smooth surface of light color, preferably a white sheet of paper, and rub the reverse of every line lightly, but evenly, with a very soft pencil or crayon, or with very fine charcoal dust. Now apply the tracing, face uppermost, to the paper on which the copy is to be made, and secure it in the proper position by means already described. Go over every line very carefully with a tracer, or with the point of a fine pen held so that the back of the nib touches the paper, pressing lightly, but evenly, drawing the instrument always towards the body, except when ruling, and keeping the hands and arms from accidentally pressing on the lines. The result will be a faint copy of the tracing, which may then be inked at once, or, should it need retouching, it may be pencilled before being inked. Owing to the liability of tracing paper to “creep” slightly in the direction in which a line is drawn on its surface, this method should not be applied to large

surfaces, except by subdividing them into squares or sections (*Par.* 45), and tracing each section separately.

50. *With the aid of Gelatine Paper.**—"A piece of so-called gelatine paper is laid over the original design, and the outlines are traced with a fine and very sharp needle (finest sort of lithographer's needle). Since the transfer thus taken must be reversed, in order to permit its copying in the manner hereafter described, the student should turn the gelatine paper and trace the outlines over again on the reversed side." "This last sketch is then rubbed over with the aid of a piece of soft leather, dipped in color-powder or lead dust; the surplus lead is then removed, and the gelatine paper now placed upon the sheet, and finally printed off by pressing carefully with a polishing steel. The perfection of the copy particularly depends on this operation." "During the process of transferring, the sheet should rest upon a glass or metal plate; a support yielding to pressure would inevitably tear the gelatine."

51. *With the aid of a Copying-glass.*—By this method the finished drawing is made by tracing directly from the original. The latter rests on a heavy plate glass, and the necessary transparency is secured by throwing a strong light on the back of the glass from an inclined mirror. The thinner grades of drawing paper are more suitable for this process, yet moderately thick paper may be used with a very strong light. The process is inapplicable to drawings mounted on muslin.

52. *By the aid of Photography.*—Photography can be employed for the reproduction of maps by printing directly from the negative, but the results are not satisfactory when fine lines are to be represented. A more satisfactory way of employing it is to make a photograph of the map on glass, and to use this, instead of the original, for tracing the copy in connection with a system of illumination similar to that explained in the preceding paragraph. Defects in the photograph, such as the weakening or obliteration of very fine lines or marks, or the accidental thickening of others, can be corrected by a careful comparison with the original drawing as the work progresses.

53. *REDUCING DRAWINGS.*—There are three general methods of accomplishing this.

* Enthoffer's Manual of Topography; D. Appleton & Co., New York.

With the aid of Auxiliary Squares, that is to say, by the subdivision of both the model and copy into squares, the sides of which must be to each other in the ratio of the reduction. The sides of the squares and other distances are laid off either with two different scales or with the aid of proportional dividers (*Fig. 13*). In other respects the process is similar to that given

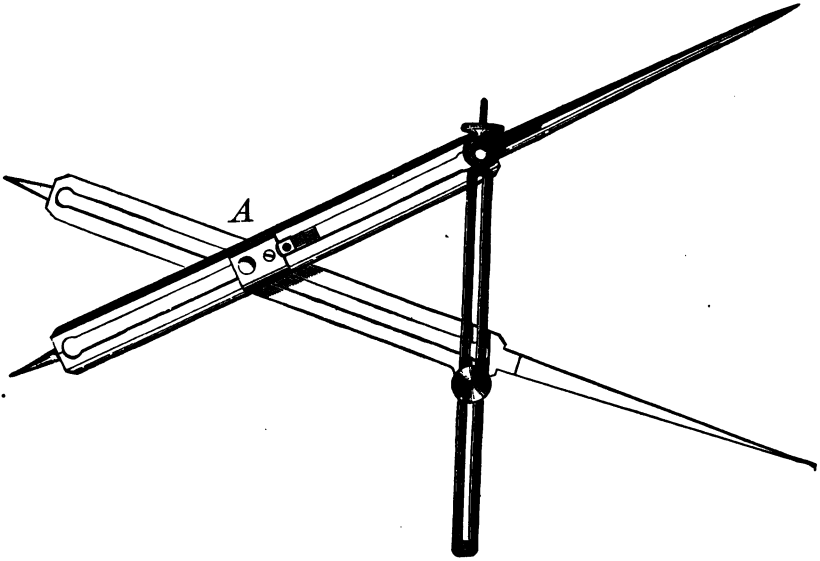


FIG. 13.

for copying maps (See *Par. 45*). The proportional dividers have graduations on the sides, by means of which, together with the slide and clamp at A, any distance spanned by the long points can be made to bear any required ratio to the span of the short ones.

54. *By means of a Pantograph.*—The general principles of construction of the pantograph can be inferred from the arrangement of the bars in *Fig. 14*, which represents a very costly form of instrument. The essential conditions to be secured are, 1°—that three points: the tracing point, the pencil, and the centre of the instrument, shall always be in a straight line during all lateral movements of the bars; and 2°—that the ratio of the distances of the tracing point and the pencil from the centre shall remain constant for any particular setting, throughout all movements of the tracing point over the paper. In the instrument here represented, the

fixed centre is at F ; but it might have been placed at any other point which would fulfill the above conditions. It is held firmly

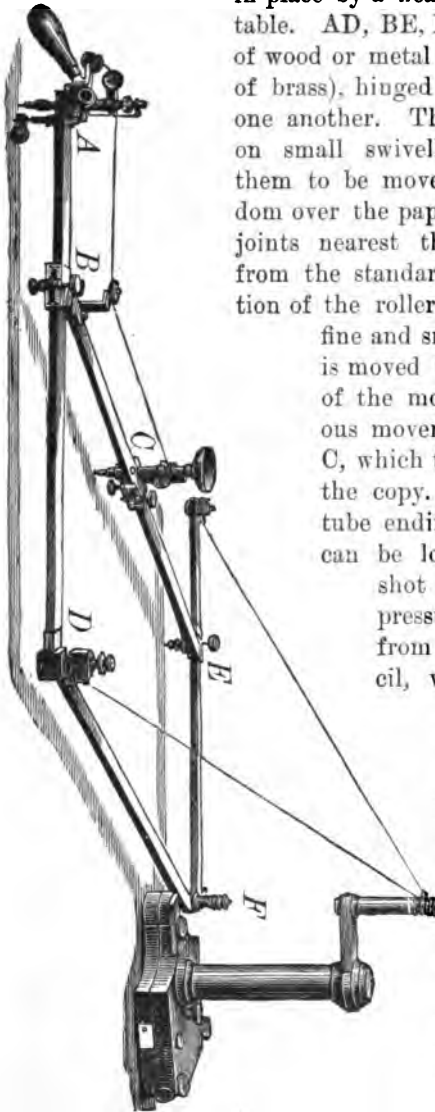


FIG. 14.

position ; and great care should be observed to adjust them in

in place by a heavy base which rests on the table. AD, BE, EF and FD are bars either of wood or metal (in this case, square tubes of brass), hinged at their connections with one another. They are usually supported on small swivelled rollers which enable them to be moved with comparative freedom over the paper ; but in the figure, the joints nearest the centre are suspended from the standard, thus avoiding the friction of the rollers and swivels. At A, is a

fine and smooth tracing point which is moved by the hand over the lines of the model, causing a simultaneous movement of a pencil point at C, which thus describes the lines of the copy. The pencil is held in a tube ending above in a cup, which can be loaded with sufficient fine

shot to secure the requisite pressure ; and a cord extends from C to A, whereby the pencil, whenever necessary, may be lifted from the paper. Graduations extending along the bars, and appropriately marked, and still finer divisions on the slides, provide for the adjustment of the instrument with great nicety to any required scale.

55. In using a pantograph, the model and copy should be brought into convenient proximity, if necessary, by super-

such relative positions that their border lines, or any other of their prominent lines, which will be designated in this connection as "guide lines," shall correspond in position, or be parallel. The pantograph, with its slides properly set and clamped, can then be applied, and the tracing point A made to coincide with the intersection of two of the guide lines on the model. Holding the tracing point securely in this position, the loaded centre of the instrument is then shifted until the pencil point C coincides with the intersection of the corresponding guide lines on the copy. When this is accomplished, the drawing may be begun ; but it is always better first, to test the accuracy of the adjustment with reference to other points on the drawings.

56. *By the aid of Photography.*—The necessary reduction is accomplished in making a photograph of the drawing on glass, which then serves as a guide or model for the draftsman, and from which a copy can be traced directly, as already described. The application of photography to the reduction of drawings has been greatly perfected in the office of the United States Coast Survey, where it is used in the preparation of models from which the maps or charts are engraved (See Coast Survey Report, 1860).

57. THE ENLARGEMENT OF MAPS can be accomplished by similar methods to those given for reducing maps ; yet all but the last are open to the objection that they exaggerate instrumental errors and the inaccuracies of the eye and hand. They cannot, therefore, be recommended when great accuracy is required.

58. THE PLATTING OF MAPS.—This consists, essentially, of laying down on paper the measured or calculated distances and angles of a survey in the same relative positions that they have occupied in the field. The line drawing, thus made, then serves as a guide or reference whereby the topographical features situated within these lines can be properly located and drawn.

Different methods of platting the notes of a survey may, very properly, be employed in constructing different parts of the same map. The proper method to be pursued in each case is to be determined by the importance of the lines and the degree of accuracy which is required ; the general rule being, that the principal lines of a survey should be platted by the most accurate method which is applicable, while methods inferior in accuracy, but more expeditious, may be used in laying down lines of lesser importance, such as "meander lines," etc.

59. The methods of platting most generally used are,

1°, *By Progression*, that is, by laying down in successive order the lines and angles as they have been taken in the field. According to this method, the line whereby the survey has been *oriented*, that is, the magnetic meridian, or the true meridian, or any established line which may have been chosen for reference, is first drawn through the point selected to represent the initial point of the survey. The angle which the first course makes with this line is then platted, and the length of the course laid off by scale. If the survey has been made by bearings, a meridian, parallel to the first, is drawn through the outer extremity of the first course and from this point as a centre, the bearing and length of the second course are platted. Similarly, the third course is platted from the extremity of the second; the fourth course, from the extremity of the third; and so on.

60. If the survey is a closed one, that is, if it returns to the point of beginning, and has been accurately made, the plat should close also. If it does not, the platting should be repeated until either a closure is effected or the error, *i. e.*, the distance between the first and last points of the plat, has been so far reduced, that it may be safely distributed in proportion to the lengths of the courses. The correction for each course, or the distance through which its terminal point must be shifted along a line parallel to that which would join the first and last points of the plat, may be found from a very simple proportion. It may also be found graphically, from similar triangles; but, usually, if the survey is accurate enough to justify its being platted at all, a careful draftsman will have so little error to distribute, that the corrections can be safely estimated by the eye. In platting a series of courses which neither close nor are connected with known points on neighboring lines, repetition is the only means of testing the accuracy of the work, unless accurate and properly selected check-bearings have been taken during the survey.

61. When the directions of the courses have been determined by measuring the angles which they make with each other, as in surveys of routes, etc., the method of platting by *Progression* would consist of merely reproducing on a small scale the lines and angles as they were measured in the field; but it is not advisable to employ it in this connection, owing to the fact that a mistake in laying off the direction of any one course tends to

displace all succeeding courses from their true position, in proportion to their distance from the source of error.

62. The difficulty just noted may be avoided in a considerable degree by employing either a natural or an assumed meridian, and by platting the courses therefrom as described in Par. 59. The assumed meridian may be the line of reference spoken of in the same paragraph, or it may be one of the lines of the survey. The angle which each course makes with this line, termed the *false bearing* of the line, in contradistinction to the magnetic and true bearings, is found by a very simple calculation.

63. The method of Progression is unsuitable when great accuracy is required, owing to its tendency to cause every error to affect all the work which follows. It should not be used, therefore, in laying down the principal lines of a map; but, if applied with proper care, it may be employed in platting lines of minor importance, "filling in," etc.

64. 2°, *Platting by Latitudes and Departures*.—These may be employed directly, by constructing at the extremity of each course, taken in regular order, a right-angled triangle whose hypotenuse is the length of the next succeeding course, and whose base and altitude are its departure and latitude respectively. But the most accurate method of platting is by the aid of a table of total latitudes and departures, which can be prepared as follows: Rule three columns, and insert in the first, in regular order, the numbers or letters designating the corners, stations, or intersections of the courses; the other two columns are for total latitudes and total departures. In the column of total latitudes, write zero opposite the first corner, or the point of beginning. The total latitude for the second corner will be the latitude, positive or negative, of the first course. The total latitude of the third corner will be the latitude of the second course added (algebraically) to the total latitude of the second corner; and, in general, *the total latitude of any corner will be the algebraic sum of the latitude of the course leading to it and the total latitude of the preceding corner*. The total departures are calculated in a similar manner. In closed surveys, the verification of the work is obtained by ending the calculation with the total latitude and departure of the first point, when, if these are found to be zero, the computation is correct.

Having prepared this table, draw a meridian through the point

selected to represent the first corner. Lay off on the meridian (always measuring from the first corner) the total latitudes in succession, and through the outer extremity of each total latitude draw a line perpendicular to the meridian equal to the corresponding total departure. The extremity of this line will be the corresponding corner, which must then be appropriately marked. Connect consecutive corners by right lines; these will be the courses of the survey.

The advantages of this method are its accuracy, ease, and rapidity.

65. Other methods of platting, viz., by Radiation, Intersection, etc., are determined by the particular manner in which the field notes have been taken. Their appropriate place is in platting the details of a map, or in the graphic representation of special problems.

66. **THE POSITION OF THE MERIDIAN.**—It is more natural, owing to early training, to view maps from a southerly point of view; and hence, the north end of the meridian is usually pointed towards the upper border of the map. Convenience, necessity, or an endeavor to secure a tasteful arrangement of the map with its title and other accessories, will determine how far the meridian need be deflected to the right or left; but, when the drawing admits of it, a close approximation to the upright position is preferable. These remarks do not apply to assumed meridians.

67. In platting, many lines are necessarily pencilled upon the paper which must afterwards be erased, and the consequent injury to the surface is often very considerable. It may be avoided by constructing the plat on another sheet of the same kind of paper as that designed for the finished drawing, and, afterwards, transferring it to the latter by pricking through the ends of the lines with a fine needle point, or by tracing.

CHAPTER V.

THE PROJECTION OF MERIDIANS AND PARALLELS OF LATITUDE.

68. The methods of platting given in the preceding chapter are suitable only for very small areas and for lines which are quite short, or which, though of considerable extent, may have been measured by means which cannot insure extreme accuracy. For the correct representation of an extensive territory a more appropriate mode of platting, viz., by *Meridians* and *Parallels of Latitude*, must be employed. These lines, having been drawn on the paper, then serve as lines of reference from which points are platted within the small figures thus formed. But the surface of the earth, being spheroidal, does not admit of being *developed*, that is, rolled out in exact dimensions on a plane surface, and hence, also, meridians and parallels cannot be drawn on a flat sheet of paper in their true relations. Various expedients have therefore been devised for projecting these lines approximately. The method to be adopted in any particular case will depend on the purpose for which the map is designed, the extent of area covered, and the degree of accuracy required.

The following are the methods most generally known :

69. THE RECTANGULAR PROJECTION.—In this the convergence of the meridians, as they recede from the Equator toward the Poles, is disregarded. This projection is, therefore, suitable only for small areas ; if applied to large ones, the scaled distances between meridians near the upper and lower borders of the map will differ too greatly from the corresponding distances on the ground. The extent of territory to which this projection can be applied is to be determined by the accuracy required.

70. *The amount of error* which will be involved in any particular case can be calculated, approximately, from the table given in Appendix C. Thus, for latitude 42° , the convergence of the meridians in an area ten miles square will be about 117 feet.

71. The errors of this kind of projection are usually distributed

among the four quarters of the sheet, as follows : Draw a central meridian, and through its middle point, and at right angles to it, draw also a parallel of latitude ; this will be the middle parallel. Divide each of these lines by scale, beginning at their intersection, into spaces corresponding to minutes of latitude and longitude respectively. These divisions can be derived for any given latitude from the table ; for latitude 42° , for example, a degree of longitude, according to the table, is 82841 metres, or 51.473 statute miles, and hence the length of 1 minute of longitude will be

$$\frac{82841}{60} = 1380.7 \text{ metres,}$$

or

$$\frac{51.473}{60} = 0.8579 \text{ statute miles.}$$

Through the points of division thus found, draw parallels and meridians ; they will be straight lines throughout their entire lengths, and will intersect at right angles. Every other right line of the map, however determined, will make equal angles with the different meridians.

72. When meridians and parallels are to be projected upon a map, one point of which is given in latitude and longitude, and when the direction of the meridian through that point is known, draw through the given point, *a*, (*Fig.* 49, *Pl.* VIII.) a meridian SN, and at right angles to it, through the same point, a straight line for a parallel, and from these two lines lay off other meridians and parallels.

Suppose, for example, the longitude of the point *a* to be $4^{\circ} 23'$ West, and its latitude, $42^{\circ} 18'$ North, its meridian being in the direction SN, and that meridians and parallels for every tenth minute are required. The first required meridian West of *a* to be determined is that of $4^{\circ} 30'$, which is seven minutes West of *a*. The first one to be determined on the right is that of $4^{\circ} 20'$, which is three minutes East of *a*.

If the scale of the survey is in statute miles, the length of a degree of longitude in latitude 42° will be 51.473. The distance from *a* to the first meridian on the left, which is seven minutes West of *a*, is therefore

$$\frac{51.473}{60} \times 7 = 6.0052 \text{ miles,}$$

which must be laid off from a to the left, and the meridian drawn through the point b , so found, parallel to NS. This will be longitude $4^{\circ} 30'$. To find the distance from a to the first meridian on the right, which is three minutes East of a , we have, similarly,

$$\frac{51.473}{60} \times 3 = 2.5736 \text{ miles}$$

This being laid off from a to the right gives at c the meridian for longitude $4^{\circ} 20'$. Other meridians are drawn parallel to NS, and at a distance apart equal to $b c$.

The parallels of latitude for every tenth minute are determined in a similar way.

NOTE.—It should be stated that, although intervals of ten minutes between these lines have been used for illustration in order to conform to the figure, intervals of one minute would be more suitable in practice.

73. **TRAPEZOIDAL PROJECTION.**—In accurate work this is applicable to areas not exceeding 25 or 30 miles square. The convergence of meridians is taken into account. The lines are straight as in the preceding case, but their intersections, except those on the central meridian, are no more right angled.

Construction.—A central meridian having been drawn and divided as before, select two points of division about midway between the middle of the map and its upper and lower borders, and draw through these points two perpendiculars to the meridian. Divide each of these lines, beginning at the meridian, into spaces representing minutes of longitude corresponding to the latitudes of these parallels, and through the points thus found draw straight lines; these will be the meridians.

74. *The errors* of this kind of projection arise chiefly from drawing the parallels as straight lines. They may be partially remedied in the following way:—The central meridian having been divided as already explained, choose, as before, two of the points of division, and draw through them short perpendiculars on either side of the meridian, each equal by scale to half the interval between the required meridians.

Let e , d , and h , g , (Pl. VIII. Fig. 50) be the points thus located; $e h$ and $d g$ will be two of the required meridians. Take the distance $e g$ as a radius, and with h as a centre sweep the arc $l m$;

then with same radius and d as a centre draw no . Likewise, with $d h$ as a radius (which should equal $e g$) and the centres g and e , draw $i k$ and $p q$. From the points e and d , and with a radius equal to $e d$, draw intercepting arcs at s and r , and find, similarly, the intersections at t and u , with $h g$ as a radius. Draw su and rt through the intersections thus found; they will be meridians, and go , uh , se , and dr , will be parallels of latitude. Other meridians can be drawn in the same manner from the points r , s , t , u , as centres. The divisions on the central meridian can then be laid off on the other meridians, and the parallels between them drawn as in the figure. In using this method it should be remembered that, while it is possible that errors of construction in projecting successive meridians may tend to balance each other, they are more likely to be cumulative, and should be carefully guarded against.

75. The figure (50) was drawn more particularly to illustrate the projection on a map of meridians and parallels of latitude from a point a , the geographical position of which has been determined. NS is the direction of the meridian passing through this point. The lines ed and gh are drawn perpendicular to NS , and the distances $e N$, $d N$, $h S$, and $g S$, are proportional to the differences of longitude of the several points. In other particulars the construction is the same as above.

Instead of the straight portions of the parallels of latitude, curves may be drawn through the intersections.

76. THE CONIC PROJECTION.—In this kind of projection the earth is conceived of as being enveloped in a tangent cone, the circle of contact being the middle parallel of the region to be represented; and the surface of this cone is assumed to coincide with that of the earth throughout the whole extent of the map. In developing this cone, parallels of latitude will appear as arcs of concentric circles, the radii of which are the elements of the cone, the latter being also the meridians of the map.

77. *Equations.*—Supposing the earth to be a sphere, as may be done in mapping comparatively small portions of its surface, the mathematical relations will be expressed by the following equations:—

Let R represent the mean radius of the earth, D , any difference of longitude in degrees, etc., and L , the latitude of the middle parallel of the map, then

The length of the arc on that parallel corresponding to the given difference of longitude will be

$$l = \frac{D}{180} \pi R \cos L \quad \dots \quad (1)$$

The slant height of the cone, or the radius of the developed middle parallel, will be

$$r = R \cot L \quad \dots \quad (2)$$

and the angle at the vertex of the cone subtended by the arc l ,

$$\theta = \frac{180 l}{\pi r} = D \sin L \quad \dots \quad (3)$$

78. *Construction.*—If, now, it were possible to plat the apex of the cone in proper relation to the map, the following would be a very simple mode of projecting the meridians and parallels of latitude:—Divide the central meridian, from the middle outward, according to the table, and extend it to a distance from the middle point equal to r (eq. 2), thus locating the apex of the cone. With the radius r , and the apex as a centre, draw through the several points of division, arcs of circles extending from one border of the map to the other. These will be the required parallels of latitude. Lay off on the middle parallel, eastward and westward from the central meridian, spaces corresponding to degrees, or smaller divisions of longitude, as the case may require, and through these points of division and the apex of the cone draw straight lines. These will be the meridians required.

79. But it is rarely possible to connect the map with the apex of the cone. In such cases, therefore, the points of intersection (and other points, if needed) on the middle parallel must be platted by rectangular co-ordinates from the middle point as an origin.

Let x be the vertical, and y , the horizontal co-ordinate of any point of intersection, then from the properties of the circle

$$x = r \sin \theta, \text{ and } y = r \text{ versin } \theta \quad \dots \quad (4)$$

Calculate from equations (4) a series of values of x and y corresponding to the differences of longitude between the central meridian and each of the others. Lay off on the central merid-

ian, from the origin upwards, a distance equal to the value of x found for the first meridian, for example, and from its extremity, lay off on perpendiculars to the right and left the corresponding value of y . The points thus determined will be the intersections of the middle parallel with the meridians immediately adjoining the central one. Other values of x and y taken in pairs, and similarly applied, will give the intersections of the same parallel with other meridians. A curve drawn through these points will be the middle parallel. Construct one of the extreme parallels by the same method, using new values of r , x , and y ; then draw straight lines through corresponding points on both parallels; these will be the meridians. The other parallels of latitude are determined by laying off on each meridian the distances already marked off on the central meridian, and drawing curves through the points of division.

80. A speedier, though less accurate, method of obtaining the directions of the meridians on either side of the middle one is to take alternate points of intersection on the middle parallel as centres, and to describe intersecting arcs of equal radii above and below the middle parallel. Straight lines passing through these intersections will also pass through the points on the parallel midway between the chosen centres, and will be the meridians. The rest of the construction is as already explained.

81. *The Errors* of the conic projection arise from the slight departure of the conical surface from the actual surface at all points except those on the middle parallel. Degrees of longitude on the upper and lower halves of the drawing are, therefore, too great, and the area represented by the map is in excess of the actual area. The method is suitable, however, with good plating, for the representation of surfaces several hundred miles in length and breadth. The map should be platted from computed latitudes and longitudes.

82. **MERCATOR'S CONIC PROJECTION**, usually known as **DE L'ISLE'S PROJECTION**, is a modification of the conic projection by the substitution of a secant for a tangent cone. The secant cone is assumed as cutting the surface to be mapped in two parallels, each of which is approximately midway between the centre of the region and its northern and southern boundaries. Between these parallels, the surface of the cone is within (below) the earth's surface, while beyond them, the cone is exterior to the

earth ; the general result being a closer proximity of the two surfaces than in the preceding case and, therefore, greater accuracy.

83. *Equations.*—The following equations express the mathematical relations, the earth being regarded as a sphere.

Let L_a and L_b be the latitudes of the lower and upper parallels of intersection respectively ; ΔL , the difference of their latitudes ; and R , as before, the mean radius of the earth. Also let r_a and r_b be the radii of these arcs when developed on the plane of the paper, then

The true length of the meridian between the two parallels of intersection is

$$\frac{\Delta L}{180} \pi R.$$

The radii of curvature of the two parallels are

$$R \cos L_a, \text{ and } R \cos L_b.$$

The lengths of the parallels for any difference of longitude, D , are

$$l_a = \frac{D}{180} \pi R \cos L_a,$$

and

$$l_b = \frac{D}{180} \pi R \cos L_b,$$

and the elements of the secant cone, or the radii of the developed parallels, are

$$\left. \begin{aligned} r_a &= \frac{\pi R (\Delta L) \cos L_a}{180 (\cos L_a - \cos L_b)} \\ r_b &= r_a - C \end{aligned} \right\} \dots \dots \dots (5)$$

wherein C is the length of the meridian between the two parallels of latitude A and B . When the extent of the survey in latitude is comparatively small, the arc of the meridian can be assumed to coincide with its chord, and

$$\left. \begin{aligned} r_a &= R \frac{\cos L_a}{\sin \frac{1}{2} (L_a + L_b)} \\ r_b &= R \frac{\cos L_b}{\sin \frac{1}{2} (L_a + L_b)} \end{aligned} \right\} \dots \dots \dots (6)$$

The angle at the apex of the cone subtended by the developed arcs l_a and l_b is

$$\theta = \frac{180}{\pi} \frac{l_a}{r_a} \dots \dots \dots (7)$$

84. *Construction.*—The values $x_a = r_a \sin \theta$, $y_a = r_a \text{versin } \theta$, and similar ones with the subscript b , will furnish all that is necessary for projecting the parallels of intersection, remembering that y_a must be laid off from the middle point of parallel A, and y_b from the middle point of parallel B.

The points having been platted, straight lines drawn through points having the same longitude will be the meridians of the map. The other parallels of latitude are platted as already described in *Par.* 79.

85. BONNE'S PROJECTION (PROJECTION DU DÉPÔT DE LA GUERRE) represents parallels of latitude by concentric circles, and meridians by curved lines concave towards the central meridian. The centre of arcs of longitude is situated on the prolongation of the central meridian and at a distance from the centre of the map [see eqs. (14) and (17), Polyconic projection],

$$r_1 = \frac{a \cot L_1}{(1 - e^2 \sin^2 L_1)^{\frac{1}{2}}} \dots \dots \dots (8)$$

in which L_1 is the latitude of the centre of the map, a , the major axis, and e , the eccentricity of the terrestrial spheroid.

86. *Construction.*—The central meridian is divided by the aid of tables (prepared from a general formula which will not be given here), and the parallels may then be swept in from the centre previously spoken of, when, after dividing the parallels, curved meridians can be drawn through corresponding points. But the intersections of each parallel of latitude with the meridians are more usually platted by co-ordinates, as already explained.

The points of division on the meridian may be taken as separate origins for the parallels passing through them, or, preferably, the curves may all be platted from the same origin, viz., the middle point of the central meridian, the axes being that meridian and a right line perpendicular thereto. In the latter case the expression for x remaining unchanged, that for y will be

$$y = c \pm r \operatorname{versin} \theta = c \pm x \tan \frac{\theta}{2} \quad (9)$$

where c is the distance from the origin to the intersection of the central meridian with the particular parallel which is being platted.

The value of r , having been found for the parallel passing through the origin, the latitude of which is L , eq. (8), other values of r are found from the expression

$$r = r_1 \pm c \quad (10)$$

c having the same signification as in eq. (9).

And finally, if n be the difference of longitude of consecutive meridians, in degrees, then, using previous notation, the length of the corresponding arc will be

$$l = \frac{\pi}{180} n N \cos L \quad (11)$$

in which N is the length of the normal to the spheroid at latitude L , and $N \cos L$ is the radius of curvature of the parallel.

But the length of the developed arc on the drawing is

$$l = \frac{\pi r}{180} \theta \quad (12)$$

Equating (11) and (12), we have

$$\theta = \frac{n N \cos L}{r} \quad (13)$$

from which the values of x and y can be calculated.

The intersections of the meridians and parallels having been platted, those lines can then be drawn through corresponding points.

87. THE POLYCONIC PROJECTION (U. S. Coast Survey) is based on the hypothesis that every parallel of latitude may be viewed as the arc of contact of a separate tangent cone with the earth's surface. There will thus be as many cones to be considered as there are parallels to be projected on the map. The apices of the cones will fall at different points on the earth's axis produced, and will approach nearer and nearer to the pole, as the latitudes of the parallels increase. The radii of the developed

parallels will therefore become shorter as the latitude increases, and the developed parallels must necessarily be divergent arcs as they recede from the central meridian.

88. *Construction*.—In applying this method on the Coast Survey, a central meridian is drawn, and carefully divided either from computed values of arcs on the meridian, or with the aid of tables (see Appendix C). Each parallel of latitude is constructed by points laid off from its intersection with the central meridian as origin,* the meridian being the axis of y , and finally meridians and parallels are drawn through corresponding points.

89. *The Formulæ* are,†

Length of normal to any parallel of latitude, L ,

$$N = \frac{a}{(1 - e^2 \sin^2 L)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad (14)$$

Radius of curvature of the meridian,

$$R_m = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 L)^{\frac{3}{2}}} \quad . \quad . \quad . \quad . \quad (15)$$

Radius of curvature of any parallel,

$$R_p = N \cos L \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Radius of developed parallel, or the side of the tangent cone,

$$r = N \cot L \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Let n be the difference of longitude of any meridian and the central one, and let θ be the corresponding angle at the apex of the tangent cone, then

The length of an arc of n degrees of longitude, and the length of the same arc developed, are given in equations (11) and (12) of *Par.* 86. Combining these values we have, as before,

$$\theta = \frac{n N \cos L}{r}$$

* Tables of co-ordinates have been prepared by Professor J. E. Hilgard, Assistant (now Superintendent) U. S. Coast and Geodetic Survey. See Report, 1859.

† The notation is that used in Professor Hilgard's paper (see C. S. Report, 1859).

Substituting the value of π from (17), we have,

$$\theta = \frac{n N \cos L}{N \cot L} = n \sin L . \quad (18)$$

The co-ordinates of points on the parallels of latitude are then, as in previous cases,

$$x = r \sin \theta, \quad y = r \operatorname{versin} \theta = x \tan \frac{\theta}{2}.$$

PART II.

TOPOGRAPHICAL DRAWING, LETTERING ; FIELD-SKETCHING.

CHAPTER I.

CONVENTIONAL SIGNS.

90. *General Instructions.*—Most of the conventional signs in pen topography are executed with the free hand, that is, without a ruler ; and a great deal of the beauty of a map depends on the execution of this, the more difficult, part of mapping. It is very important, therefore, to take advantage of every position of the hand and of the paper which will facilitate the drawing of clear, firm, and steady lines. The penholder is held with a short hold between the thumb and the first and second fingers, the latter being dropped below the holder until it nearly touches the paper, while the other fingers serve as a rest for the hand. Some draftsmen use the second finger for a rest, allowing it to touch the paper very lightly. Both points of the pen should press equally on the paper ; if held slanting sidewise, it will make a scratching noise and will be likely to trace a ragged line. As already explained, the pen should be drawn towards the body ; and this should be done with great steadiness, drawing the line of full width at one stroke, and making it perfectly square at both ends, excepting where it is required to be otherwise by the particular sign which is being represented. Should the line be long, the stroke must not be carried so far as to cramp the fingers, for that would soon tire the hand and render it unsteady. In such a case, the pen should be lifted quickly from the paper, and the hand moved lower down ; the line can then be continued, taking care, however, to render the joints in the line invisible. The ink on the pen thickens very rapidly, and, therefore, has to be wiped off quite often. Although this is a source of annoy-

ance and delay, it must not be neglected. A soft piece of linen and some water are needed for this purpose.

91. *Cleared land, grass, or meadow* (Pl. I., *Fig. 1*), is indicated by groups of short lines arranged like tufts of herbage. Each of these little figures is drawn with its base always parallel to the base of the drawing, whatever may be the shape of the inclosure containing this sign. The better to secure this parallelism, guide lines should be drawn in pencil, here and there, over the plate. The tufts must be spread evenly over the paper, not too thickly; and all appearance of regularity or approach to arrangement in lines must be avoided. Each tuft is composed of from five to seven lines, drawn as if converging into a point below the base, as is shown on a large scale at A,—the middle lines being the longest, and the extremes mere dots. The base must be a straight line, and not curved as at B or C.

It is recommended that this sign be the last one drawn on a map. If well executed, its general effect when viewed from a little distance is that of a flat tint; which if it be made to harmonize in tone with the rest of the drawing, by a judicious selection of the sizes and spacing of the tufts (and even of the shade of the ink, if necessary), will add materially to the beauty of the map. The course recommended is also the best means of securing uniformity of style and effect in the execution of this particular sign.

92. *Cultivated land* is represented by alternate broken and dotted lines, suggesting furrows. These are, for the sake of variety, made to assume different directions; one set of them being, generally, parallel to one of the sides of the inclosure (Pl. I., *Fig. 2* and *12*). The ground must first be prepared by drawing light pencil lines equidistant from, and parallel to each other, in sets. These pencil lines must be ruled; and if they cannot be spaced off by the eye, their intervals must be determined, and marked by dots. Then draw the broken and dotted lines, by hand, over the pencilled guide lines. The joints in the broken lines must not be opposite to each other, and the breaks, or vacancies in the lines, must be very short. The dots of the dotted lines must be made by touching the point of the pen to the paper and immediately lifting it off, without dragging it over the paper; this will make a round dot. The dots must be fine, and close together. See plate of Conventional Signs and

Symbols for the mode adopted by the Coast Survey of representing this feature.

93. *Trees, woods, etc.* (Pl. I., *Fig. 3 to 8*).—A tree, being one of those features whose conventional sign is intended to suggest, in some degree, the object itself, is represented by characteristic lines resembling a pen drawing of a tree as seen in horizontal projection, with its shadow upon the ground cast by parallel rays inclined 45° to the horizon.

Trees, as to arrangement, are represented as single trees, or in groups, or in masses. As to kind, the only distinction which seems necessary for general purposes in this country is that between *deciduous trees* (*Figs. 3 and 4*), and *evergreens* (*Fig. 6*); the former, when viewed in plan, being characterized by a rounded outline; the latter, by a star shape. A separate sign (*Fig. 3, c*) is sometimes used, however, to represent oak woods. The corresponding sign adopted by the Coast Survey is shown on the plate of Conventional Signs.

The sizes of the trees may be indicated approximately, where the scale of the map admits of it, by enlarging or reducing the particular sign used.

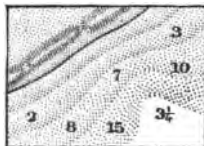
94. Pl. I, *Fig. 3, A*, shows a mode of representing deciduous groves or open woods which is used to a considerable extent by civil engineers. The sign consists of scattered clusters interspersed with single trees of different sizes; the intent being merely to indicate the character of the growth, and not to represent the particular arrangement. At B, is another and more natural mode of representing the same features, which may also be adapted to the representation of dense woods by the introduction, here and there, of heavier masses. The outlines, to have a good effect, should be made with simple curves, firmly drawn, and should not be cut up by very small indentations. The trees and clusters must be disposed in every possible variety of position, avoiding carefully all arrangement in lines or in regular figures.

95. *Orchards* are shown by placing single trees with their shadows upon the intersections of two sets of equidistant parallel lines drawn at right angles with each other, as in *Fig. 4*. One set of these lines is usually placed in the direction of one side of the inclosure. The lines are drawn in pencil, and are afterwards erased. Single trees are drawn, as shown on a large scale

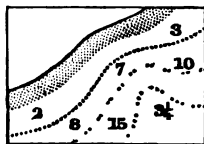
Conventional Signs and Symbols

<i>Flashing Light</i>	✱
<i>Fixed Light</i>	✱
<i>Light Vessel</i>	✱
<i>Lights (Small Scale Chart)</i>	•
<i>Old Light Tower</i>	•
<i>Buoys (Can, Nun, Spar, and Bell)</i>	
<i>Green or Red Buoy</i>	○
<i>Black</i>	●
<i>Horizontal Stripes Buoy</i>	▬
<i>Perpendicular</i>	▬
<i>Whistling</i>	⚡
<i>Lighted</i>	⚡
<i>Mooring</i>	⚡
<i>Beacon (not lighted)</i>	▲
<i>Spindle or Stake</i>	▲
<i>Bell Boat</i>	⚡
<i>Wreck</i>	✱
<i>Life-saving Station</i>	⚡
<i>Rock under Water</i>	+
<i>Rock awash</i>	✱
<i>Kelp</i>	⚡
<i>No bottom at 20 fathoms</i>	20
<i>Triangulation Point</i>	▲
<i>Dwelling House</i>	■
<i>Barn</i>	■
<i>Shed and Pen</i>	□

*Shoreline, Low Water
and 6, 12 and 18 ft.
Sanded Surfaces*



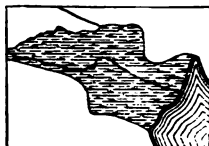
*Shoreline, Low Water
and 6, 12 and 18 ft.
Dotted Curves*



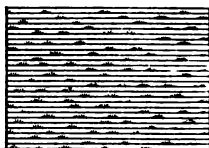
Rocky Ledges



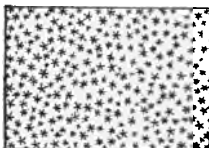
Fresh Marsh



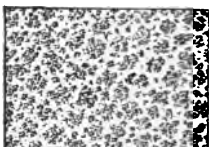
Salt Marsh



Pine



Oak



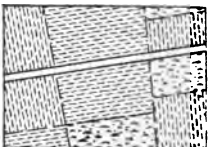
Deciduous



Live Oak



Cultivated Lands



Mud



in the left-hand corners at A and B. The shadow is detached from the outline of the tree, and is intended to have an elliptical form. When the scale is small, a single tree becomes a simple circle shaded with the pen on the lower and right-hand sides, and the shadow may, with advantage, be omitted. (See *Fig. 12.*) *Fig. 4* illustrates two different styles of finish in the representation of orchards. That illustrated on the left, though somewhat more laborious, produces a richer effect, and, by the contrast of light and shade, suggests the idea of relief.

When trees occur upon a hill-side, they must be more sparsely distributed than on a plain, in order to interfere as little as possible with the hill-shading. The shading lines of the hill-side are interrupted, in order to receive the body of the tree, but not its shadow, which may be drawn independently of them when the slope is slight; it is still better to omit the shadow altogether. When the slope is steep the shadows must be omitted; and a wood or forest should be represented as less dense than where it is drawn upon a more moderate inclination.

96. *Evergreens* are represented in *Fig. 6*, A and B, and also in the plate of Conventional Signs. The first mode of representing this feature is more ornamental than the second, and is the most suitable for larger scales, while the latter is applicable to the representation of small trees or bushes, or of large evergreens in maps drawn to small scales. The second sign (B) has been adopted for general use by the Coast Survey, as admitting of greater uniformity of execution, and as being more economic of time and more legible than the other. At C, is represented a mingling of oak and pine woods.

97. *Fig. 7* is intended to illustrate two of the numerous styles of representing clusters or masses of trees and bushes in landscape gardening. They are selected and recommended for their simplicity and the suggestiveness of their lines. The treatment at B differs from that at A, in a simpler application of curved shading lines, and in the use of parallel shading lines for the purpose of rendering the groupings more conspicuous. Shadows are inapplicable to pen and ink drawings of this kind; they only weaken the outlines of the groupings, which, except in vague sketches, should be very clearly defined. The star-shaped figures represent evergreens.

98. Single trees are sometimes represented in elevation (*Fig.*

8) in landscape gardeners' plans, partly for pictorial effect, and also with the view of rendering the different kinds distinguishable through their characteristic outlines. But this should be done very guardedly in pen-and-ink drawings, or better, perhaps, not at all. Such plans, executed in ink, are usually drawn to a small scale which does not admit of a ready discrimination between deciduous trees. Evergreens may be thus drawn in order to distinguish them from other trees, as at *c*; but, generally, it is better, as has already been intimated, to represent all trees in plan. (See *Fig. 14*.)

99. *Fig. 8* also represents at *a*, a sign for low evergreen bushes and beds of trailing evergreens. Also at *b*, a simple mode of shading the interior of flower-beds is suggested. The effect is rather more delicate if the lines are made fine throughout. Another mode of shading flower-beds is, ruling them with fine lines, as in shading a house, or with broken lines. They are also sometimes dotted, as in sanding a shore.

100. *Fig. 14* is intended to illustrate the mode of representing with the pen the arrangement of ornamental grounds. Trees of special importance, shown either singly or in groups, are distinguished by letters or numerals corresponding to those of an explanatory table, which may be either neatly printed on the margin of the drawing or prepared separately.

101. *Uncultivated land* which is not forest, but such as brushwood, heath, &c., is represented by mixing tufts of grass, and dots, with small bushes of less dimensions than those of the trees in the sign for forest. (See *Fig. 5*.)

102. *Sand and gravel* are shown by dots,—the latter rather coarser than the former. In making these, the pen should not be jerked over the paper at random, but put down and taken up slowly, and never without an *intention* in regard to the position of every dot. Make, first, a row of heavy dots along the shore line; then another row a little lighter than the first and just behind it, with the dots placed opposite the spaces between the dots of the preceding row; and so on, gradually reducing the sizes of the dots, and slightly widening the spaces; giving the surface a shaded appearance, somewhat similar to that obtained by water-lining. See Plate VI. and Conventional Signs. Sand-hills, or dunes may be represented either by making the sides of their slopes darker than the level parts,

with heavier dots placed closer together, or by drawing the conventional sign for slopes over flat sanding. See Conventional Signs.

103. *Fresh and salt marsh.*—The sign for *fresh marsh* (Fig. 9) is usually indicated by fine and full lines ruled always parallel to the base of the drawing, and grouped in an irregular manner, so as to leave small islands interspersed through it. These islands are filled with grass, with here and there, a bush or a group of bushes, if any be there. The division between the land and water should be first sketched lightly with a pencil, as a guide for ruling the lines. The texture, as to fineness, of this sign should be regulated by the scale of the map, and should be consistent with the other lines in it. The sign now used by the United States Coast Survey is shown on the plate of Conventional Signs. It is usually drawn, however, without the boundary line there represented, and is thus described by Mr. E. Hergesheimer: * “Lines of the same strength and the same distance apart as those of the salt marsh, are ruled and irregularly broken, then interlined and tufted by free hand with light short lines grouped irregularly.” The sign for *salt marsh* is indicated by Fig. 10 of Plate I. See also Plate VI. and Conventional Signs.

104. *Water.*—When the delineation of the ground extends no further than the water surface, it is customary to represent water surfaces by lines drawn parallel to the shores and graduated outwards from them. They are called *Water-lines*. In order to execute properly this style of shading water, the following directions must be strictly observed. After having drawn a moderately stout line of uniform thickness, called the *shore line*, as at A, Fig. 11, for the outline of the water throughout the whole drawing, begin by applying to it as closely as possible the first water-line. In doing this, attend to the narrow white space between the two, making it a *fine white line*, and of *even width*. The first water-line may be nearly of the thickness of the shore line, and should follow it closely in its minutest deviations. Apply such a line to every shore line in the drawing (see Fig. 12). When that is done, proceed to the second water-line, which may be a little finer than the first, and a very little

* Appendix No. 11, C. S. Report for 1879.

more removed from it than the first was from the shore line. Carry this one throughout the drawing, in the same manner as the first one. Then take up the third line, drawing it a little finer, and increasing by very little the distance between the lines. In this way, go on, adding one line at a time to every shore on the drawing, increasing very gradually their intervals, and diminishing as gradually their thickness. These lines should be drawn *clean*, and as steadily as the hand can make them. Take a very short hold of the pen, resting the point of the middle finger upon the paper. Each line should be of uniform thickness throughout its length, and kept constantly at the same distance from the one last drawn. Draw always towards the body, turning the drawing as before directed, and keep, always, the line last drawn *to the left* of the one in progress, so that the distance between it and the point of the pen may be constantly watched. The lines must be *completed* successively, as above directed, for the following reasons : 1st. The eye becomes accustomed to the interval employed, and thus the confusion attendant upon carrying on at the same time three or four lines having different intervals is avoided. 2d. By this equal distribution of the lines, symmetry is insured ; because, whatever be the width of different channels (*Fig. 12*, at B), an equal graduation of tint from every shore is obtained, and the shading lines meet in the middle, which, otherwise, might not always be the case. 3d. It enables us to conform to a principle, which is, that every line must return to itself,—no *spiral* lines being admitted. But sometimes two lines will coalesce, as at *c, c*, *Fig. 12*, where they join into one at *d*, and afterwards separate into *e, e*. The last line in the middle of a piece of water must be a line returning to itself, and not a spiral. When the shading lines meet the margin of the drawing they are cut off ; but they are drawn as if they were to be continued out of the margin. These instructions may seem over minute, but the beginner must be warned that this is the most tedious and uninteresting part of topographical drawing, and requires great care and patience in order to produce a good effect.

105. *High and low tides* are represented as in Plate VI. ; the former being defined by a sharp and full black line ; the latter, by the darker edge of the sanding which extends beyond. Different depths of water below low tide are shown by means of level

curves of stated depths. Each curve may be distinguished by a distinctively dotted line, as shown in the plate of Conventional Signs. This method is very economic of time, and although it does not produce as beautiful a drawing as that in which the curves of depth are defined by the darker edges of sanded strips which cover the intervals between successive curves (see conventional signs), it is preferable to the latter. In all these cases water-lines are omitted.

106. The high and low water margins of streams affected by freshets may be shown somewhat after the manner indicated by Pl. I., *Fig. 11*. The figure was designed, however, merely as a preliminary exercise in drawing water-lines. In such cases, having completed the water-lining between the low water shore lines, ink in lightly the features of the land which is subject to overflow, and draw over such of them as are covered by the overflow very fine water-lines, beginning with the high water shore line and ending each line abruptly in the low water shore line (*Fig. 11*). This shading should be much fainter than that of the stream, no heavy lines being used.

107. *Rivulets, or very small streams* are represented by one, two, or three lines, according to their magnitude (*Fig. 12*).

108. *Water-falls, rapids*.—*Fig. 13* is intended to represent the mouth of a lakelet discharging its waters through a rocky gorge at the head of which is a water-fall, followed by rapids. Another water-fall has been introduced in the lower part of the stream for the purpose of showing how to pass from the fall to the water-lining representing smooth water beyond it. On the left are represented mills with their feeder and tail-races.

The sign here used for a water-fall is not the one usually employed. But it is believed to be a better one and more suggestive of the object it is designed to represent. It is also adapted to the representation of dams. The sign for rapids is believed to be entirely new, and is very easily made.

109. The different forms of signs for *buildings, inclosures, roads, etc.*, are given in Pl. V., *Fig. 40*. The mode of representing some of these objects on the Coast Survey maps is shown on Pl. VI.

It must be remembered that the dimensions of all these signs are variable, and must correspond to the scale of the drawing.

CHAPTER II.

THE REPRESENTATION OF SLOPES.—THE HORIZONTAL SYSTEM.

110. TWO GENERAL SYSTEMS of representing slopes with the pen are used. They are called, respectively, the *horizontal* and the *vertical* system.

111. THE HORIZONTAL SYSTEM. *Contours*.—This consists in intersecting the inequalities of the ground by a series of horizontal planes at equal vertical distances apart, and “projecting” upon the map the curves in which these planes intersect the surface. These curves are known as hill-curves, or, more generally, contours. To explain the above process by a familiar illustration, let us suppose a hill rising out of the water, as in *Fig. 15*, where such a hill is represented *in profile*—AB being the water-surface. The edge of the water would be perfectly *level*, or, approximately, a *horizontal curve*. This water-line may then be regarded as a curve cut out of the hill by a horizontal plane, and, as such, we may measure its dimensions, determine its proportions, and draw, or “project” it on our plan.

Suppose, now, the water to rise one foot. A new curve will be defined on the hill-side at a vertical distance of one foot above the first at every point. This new curve will possess properties similar to the first one, and may, like it, be determined and projected. CD is the plane of the second curve. In the same manner, other planes, as EF, GH, IK, and LM, at the same vertical distance apart, may be conceived of, and their curves measured and drawn.

Let the curves now be projected upon a horizontal plane; that is, suppose the eye to be placed above the hill, so as to look directly down upon every point of its surface. The curves will then be drawn as in *Fig. 16* (the shading lines excepted).

112. To understand how these horizontal sections contribute to the knowledge of forms, compare *Figs. 18, 19, 20*, which repre-

sent in plan and elevation (profile) a square right pyramid, a right cone with a circular base, and a hemisphere. Each of these solids is represented as having been divided by horizontal cuts into six slices of equal thickness, the edges of which have been projected, or transferred to a horizontal surface, giving the lower parts of the figures. The sections of the pyramid will thus be shown to be in plan a series of concentric squares; while those of the cone and hemisphere are concentric circles. In the case of the cone the curves are *equidistant*, indicating a *uniform declivity*. In the hemisphere the horizontal distances between them diminish rapidly as we approach the lower curves, which indicates an *increase in the declivity* in that direction. Similarly, *Figs. 15 and 16* will show that where the curves are at uniform distances apart, the slope is also uniform; where the hill is steep, the horizontal curves are closer than where the inclination is less; and that the proximity or remoteness of the curves varies with the steepness or gentleness of the declivity.

113. Each of these contours gives us, throughout its length, an exact idea of the shape of the ground; but in the spaces between them we are necessarily left in ignorance of its precise form. When, however, we have obtained such a number of sections as will furnish a sufficiently close representation of the ground for the purposes required (and this number may be increased according to the requirements of the case), it is allowable and customary to consider the surface between any two sections as sloping *uniformly*.

114. *Horizontal zones*.—The surface between two consecutive contours is called a horizontal zone. This zone may be considered as generated by a straight line placed in the direction of the slope of the zone and touching both of the curves, and which, being kept normal* (perpendicular) to the upper curve, is moved around the hill, fulfilling constantly the above conditions, until it returns to the point whence it set out. The successive positions of this line are the elements constituting the surface of the zone. The height of a zone is the vertical distance between consecutive cutting planes; the width of the zone is the horizontal distance between its contours; and the slope of ground

* A normal to a curve is a right line, which is perpendicular to a tangent to the curve, at the point of contact.

is represented by the position of the generating line mentioned above. These three right lines constitute, at any given place, the altitude, base and hypotenuse of a right triangle.

115. Therefore, in order to find the actual inclination of a zone at any place, find the element at that place by drawing a normal to the upper curve of the zone. Construct, with the normal line mn (Fig. 16), so found, as a base, and with the height of the zone as an upright, a right triangle; its hypotenuse will be equal to the true line of the slope, and the acute angle at the base will be equal to the angle of inclination required. This triangle is seen at ACa (Fig. 15), where the angle CAa is the angle of inclination.

116. A general scale of inclinations may be constructed for all cases, as follows:—Draw the lines AO and OB (Fig. 17), forming a right angle at O . Lay off with a protractor the lines $O 5$, $O 10$, $O 15$, $O 20$, &c., making, with OB , angles of 5° , 10° , 15° , 20° , &c., successively up to 45° . To use this scale, draw the line CD parallel to OB , and at a distance above it equal, according to the scale of the map, to the vertical distance between the horizontal sections. Having found the normal at any point, take its length in the dividers, set one foot of the dividers at C , and the other towards D on the line CD . Should the length of the normal be Ce , the inclination is 5° ; should it be Cg , it is 10° ; should its extremity fall at f , midway between e and g , the inclination is $7\frac{1}{2}^\circ$. If it fall at a point not easily determined as to its position, draw a line from O through the point, and measure with a protractor the angle formed with OB .

117. A special scale of inclinations may be prepared for any particular scale of construction with the aid of a table like the following :*

* Copied from a paper by Mr. E. Hergesheimer; Appendix No. 13, U. S. Coast Survey Report, 1880.

<i>Slope.</i>	<i>Proportion of Height to Base.</i>	<i>Length of Base to 1 ft. of Height (in ft.).</i>	<i>Length of Base to 20 ft. of Height (in ft.).</i>	<i>Length of Base to 20 ft. of Height (in mtrs)</i>
1°	1 to 57	57.29	1145.8	349.1
2°	1 to 29	28.64	572.8	174.5
3°	1 to 19	19.08	381.6	116.3
4°	1 to 14	14.30	286.0	87.1
5°	1 to 11	11.43	228.6	69.7
10°	1 to 6	5.67	113.4	34.6
15°	1 to 4	3.73	74.6	22.7
20°	1 to 3	2.75	55.0	16.7
25°	1 to 2	2.14	42.8	13.1
30°	1 to 1.7	1.73	34.6	10.5
35°	1 to 1.4	1.43	28.6	8.7
40°	1 to 1.2	1.19	23.8	7.2
45°	1 to 1	1.00	20.0	6.1
50°	1 to 0.8	0.84	16.8	5.1
55°	1 to 0.7	0.70	14.0	4.3
60°	1 to 0.6	0.58	11.6	3.6

Draw two parallel lines about half an inch apart, and, beginning at the extremity of one of them, lay off in succession, by the scale of construction, distances having the same ratios to the assumed height of zone as those given approximately in the second column of the table, or, more exactly, in the third. Mark the spaces thus laid off with their corresponding inclinations taken from the first column of the table, and draw through the points of division perpendiculars from one parallel to the other. The resulting diagram will be a scale of inclinations, or of contours, for any map drawn with the same scale of construction as that used in constructing the diagram. See the upper edge of *Fig. 30*, *Pl. III*.

Example.—Let the scale of the map be 400 feet to 1 inch, or

28.64, and the height of the zones, 10 feet ; required a scale of contours.

By the table (columns 1 and 3), the horizontal distance between the curves of a slope of 2° will be $10 \times 28.64 = 286.4$ feet ; the curves of a slope of 3° will be $10 \times 19.08 = 190.8$ feet apart ; a slope of 4° will require curves 143.0 feet apart ; and so on for other inclinations. The corresponding spaces on the scale of contours will be respectively .71, .47, .36 inches, etc.

The most convenient way of using such a scale is to cut it out and apply it directly to the map.

118. *Determinate contours*.—Curves accurately determined by survey or otherwise are called determinate curves or contours. They are always equidistant vertically ; and if a sufficient number of them is used to render the assumption quite near the truth that the slopes between adjacent ones are straight, the shape of the hill represented by them will be completely defined.

119. *Pencilling contours*.—The field work whereby the location of contours is determined may be conducted in different ways, according to circumstances, but the immediate purpose of the several methods is the same, viz., to determine a large number of points on each contour, through which, when platted on the map, the contours can be drawn with a sufficiently close approximation to accuracy. The modes of platting points of contours from the notes of a survey are separately considered and explained in *Par.* 123. The curves are first drawn through these points with a pencil. The lines should be smooth and fine, and yet not so faint as to make it difficult to distinguish them. They may be drawn with the free hand or, whenever a favorable arrangement of points renders it desirable, with the aid of an irregular curve. In either case, however, the curves must follow the points with great fidelity, and should pass from one degree of curvature to another with a flowing movement, forming no angles or abrupt changes which are not indicated by the field notes (see *Pl. VI.*). If a line is accidentally misplaced, it should not be left on the drawing and the correct one traced beside it, but it should be carefully erased and then redrawn in the proper position.

120. *Inking contours*.—The contours, having been completed in pencil, are inked. This is frequently done in india ink, when a representation of the undulations of the ground is the chief object of the map. But, usually, some color is preferable which,

by contrast, will render the contours conspicuous without obscuring the other topographical features of the drawing. Carmine, crimson lake, and aniline red have been used for this purpose ;* but venetian red, burnt sienna, or raw sienna are to be recommended for general use. The inking of contour lines must be done with great care, using fine, firm lines, applied with great exactness to the pencil lines previously drawn. The mapping pen and the contour pen have each their appropriate sphere in this work :—the former, in tracing very sharp curves which cannot be conveniently drawn with the latter, while the contour pen is especially suitable, with the aid of an irregular curve, for drawing the longer and flatter contours.

121. *The reading of altitudes.*—In order to facilitate the reading of altitudes, the heights of some of the contours above the datum of the survey, selected at regular intervals, should be indicated by appropriate numerals. It is customary to interrupt a contour line at such points, and to write its height in brackets within the break. Another expedient, with the same object in view, is to make every fifth or tenth contour heavier than its neighbors (see Pl. VI.). But in drawing contours in india ink fine lines only should be used. ✓

In the application of this method of delineating hills or slopes, it almost always happens that the summit of the hill is not occupied by a determinate contour, owing to the insufficient projection of the summit above the highest determinate curve. In such cases, the top of the hill is indicated by a dotted contour within which the height of the summit above datum should be neatly inscribed.

122. With the view of exhibiting more clearly the relative inclinations of different parts of a hill, the spaces between successive determinate contours have sometimes been filled with *auxiliary horizontal curves* (see Pl. II., Fig. 16, at *e*, *o*, and *d*) ; the effect produced being that of a shaded surface which, if the auxiliary lines are sufficiently numerous, quite readily suggests the undulations of the ground. The necessary conditions of this interlining are : that the same number of auxiliary curves should be drawn in every zone of the map ; that the lines should divide the zones into strips of equal thickness ; and that the curves should

* Especially suitable, when other colors are employed to indicate the location of pipes, drains, etc.

not be so numerous as to coalesce in the steeper parts of the hill. The lines may be full lines, but finer than the determinate contours, and in that case should be continuous throughout; or they may be graduated in thickness in proportion to their proximity. In the latter case, a bolder effect is produced by making the lines occasionally broken; drawing the heavier curves with short, firm strokes about $\frac{1}{4}$ of an inch in length, and gradually making them lighter, and extending their lengths, as the slope becomes less steep.

The errors in these styles of hill-shading which are of most frequent occurrence are: the termination of the strokes of consecutive zones on the same straight line, as at *d*, *Fig. 16*; regularity in breaking the lines, as at *r*; and the overlapping of lines, as at *s*.

It must be added that the use of auxiliary curves as here described has been practically abandoned.

123. *To construct the points of contours.*—In surveying the ground for the purpose of tracing upon its plan the horizontal curves, the points whose levels are determined must be sufficiently numerous and close together, to admit, without sensible error, of the supposition that the slope of the ground between them is uniform. The following method of platting the curves proceeds upon this supposition.

Let *A* and *C* (*Fig. 51*) be two points on the profile of the ground, and let the horizontal distance (*AB*) between *A* and *C* be fifty feet. Let the difference of level between *A* and *C*, as determined by survey, be ten feet, *C* being the lowest point. It is required to find upon *AC* the points in which horizontal planes, drawn one foot apart, and commencing at *A*, will intersect *AC*. The following proportion will discover this:—

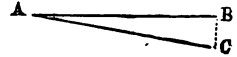


FIG. 51.

As the total fall from *A* to *C* is to the horizontal distance *AB*, so is any partial fall from *A* towards *C* to its corresponding horizontal distance from *A*.

Now, *AB* is 50 feet, and the total fall from *A* to *C* is 10 feet; then, for a partial fall of one foot, we shall have 10 ft. : 50 ft. : : 1 ft. : 5 ft., or the horizontal distance from *A* to that point of *AC* which is one foot below *A*. By laying off 5 feet from *A* towards *C*, we shall have the intersection of the one-foot plane with a line of the ground. Again, 10 : 50 : : 2 : 10, which gives

ten feet from A for the point of intersection of the two-foot plane with A C ; and so on for the other planes.

The heights of the different points in any particular survey, called their *elevations* or *references*, are all estimated from some horizontal plane arbitrarily chosen as the *datum plane* or *plane of reference*. Whether this plane be above or below the territory surveyed (the latter is the usual case), the references, as obtained by the instrument, are scarcely ever expressed in whole numbers ; and whereas it is desirable that the references of the contour planes should be whole numbers, the labor of stating a proportion to calculate every point becomes considerable. This is obviated by the following convenient mechanical method,* by which the proportions, instead of being stated in figures, are presented in lines, by means of the properties of similar triangles.

124. Let 1, 3, 9, 7 (*Fig. 52*), be a portion of ground, projected on a scale of 50 feet to an inch. It is 100 feet square, and is subdivided into four squares with sides of 50 feet. Let the references of the points 1, 2, 3, etc., be respectively 8.10, 6.30, 7.25, etc., as indicated in the figure. These levels are expressed in feet, and are referred to a horizontal plane 2.5 feet above the point 5, which is the highest point of the ground. It is required now to trace the intersections of horizontal planes which shall be 3, 4, 5, 6, etc., feet below the plane of reference. Let us begin with the line 5, 6. Draw the line A B (*Fig. 53*), equal to the line 5, 6, or, according to the scale, fifty feet in length. Then let fall from A and B two perpendiculars, A D and B C. Divide these perpendiculars into equal parts, say, each one-tenth of A B, and join the opposite points of division, forming the figure A B C D. Number the horizontal lines from B downwards, in quarters of unity, viz., .25, .50, .75, 1, 1.25, 1.50, etc., etc., so as to include the greatest descent of the ground from station to station. In the present case 7.50 will suffice. Cut from a piece of stiff paper a narrow strip like E F, making the edge E F accurately straight. Fasten the line E F to the point B, by means of a fine needle, so as to conceal as little as possible of the corner at B, and the instrument is ready for use. Beginning at the central point, 5 (*Fig. 52*), it will be observed that the

* Industrial Drawing. D. H. Mahan.

three-foot curve is .5, or half a foot below it; the four-foot curve is 1.5 feet below it; the five-foot curve, 2.5, and the six-foot curve, 3.5 below it. The total fall from the point 5 to the point 6, is 6.50 — 2.50, or 4 feet. Then the edge E F (*Fig. 53*) of the strip must be placed so that the line E F will be drawn from B to G on the horizontal line marked 4, corresponding with the difference of level between stations 5 and 6. The strip must be secured in this position by a pin near E. Now from station 5, the first partial fall we wish to find is from reference 2.50 to the three-foot curve, or .5 of a foot.

The proportion is, total fall from sta. 5 to sta. 6 is to distance from sta. 5 to sta. 6, as partial fall is to the distance required; or, by the hinged strip, A G (or B H): H G :: B i : i k.

Hence, to find the horizontal distance for the partial fall of .5, we have only to measure on the horizontal marked .50, its length included between B H and E F (*Fig 53*), and lay it off on the line 5, 6 (*Fig. 52*), from 5 towards 6. This will be a point of the three-foot curve. The next point, that of the four-foot curve, is 1.5 feet below station 5. Take the length of the line marked 1.50, included between B H and E F, and lay it off from 5 towards 6. The five-foot curve lies 2.5 feet below station 5; then we take the part of the line 2.5 included between B H and E F, and lay it off as before. The six-foot curve being 3.5 feet below, we measure and lay off a similar part of the line marked 3.5. This finishes the division of the line between sta. 5 to sta. 6, and gives points of the three, four, five, and six-foot curves, which must be marked (3) (4) (5) (6). Points of the curves on other lines are determined in the same manner. For example, from sta. 5 to sta. 4, the total fall is 6.50 feet. Set the edge E F from B to L on the line marked 6.50, and measure, and lay off successively from 5 towards 4, the parts of the lines marked .50, 1.50, 2.50, 3.50, 4.50, 5.50 included between E F and B M. The station 4, having a reference of 9 feet, is itself a point of the nine-foot curve. Find the points thus upon every line of the figure, and draw the curves through the points so determined, taking care to give them their proper curvature from point to point. If the total fall from station to station is expressed in smaller fractions than .25, as for example from sta. 5 to sta. 2, where it is 3.80; then the line E F must be placed at a point between 3.75

and 4, but nearer to 3.75; or else, the line BC may be divided and numbered so as to show smaller fractions than .25.

The horizontal distance from B towards A must always be the distance by scale between *consecutive* points the altitudes of which have been determined instrumentally.

125. The preceding demonstration can be very easily applied to the case (the usual one) where the plane of reference is below the ground surveyed. A strip of cross section paper is a very convenient substitute for the strip $ABCD$.

126. *To project horizontal curves of the ground under water.*—In surveying a harbor or any extensive body of water, flag-buoys are stationed at convenient points, and their positions, their distances from each other and from some points on the shore at the water-line, are accurately surveyed, and projected on the map. Soundings are taken along these connecting lines, at equal intervals between any two of the stations, though the intervals may vary between different stations. To determine the curves of the bottom, it is necessary to distribute the soundings of each line equally throughout its length. Suppose the line between the two buoys (*Fig. 40*, “soundings”) to be one of the projected lines, and that its length is 630 feet. The number of recorded soundings corresponding to that line is 22, including the soundings at the buoys. This will give 21 intervals between the soundings. Then the line must be divided into 21 equal parts, of 30 feet each. Mark the points of division on the line, and write opposite to each point its corresponding sounding. The points of any desired curve may now be found;—for example, the six-foot and nine-foot curves will pass through the points 6 and 9, twelve-foot and fifteen-foot curves will pass midway between the points $11\frac{1}{2}$ and $12\frac{1}{2}$, and $14\frac{1}{2}$ and $15\frac{1}{2}$, respectively. In the same manner other lines of soundings may be divided, and points of the curves determined. Through all the points so found, the curves are drawn, after which they are numbered, as in the figure, at a sufficient number of places.

127. The following easy method of dividing a line into any number of equal parts will save the labor of measuring, or dividing by trial. Cut a strip of drawing paper, the edge AB of which (*Fig. 53-A*) is graduated in equal divisions. If it be required to divide the line cd into nineteen equal parts, place the strip so

that its edge A B shall make a convenient angle with $c d$, and so that the zero of its graduation shall coincide with the point c . Secure the strip in this position. Now join the point d with the

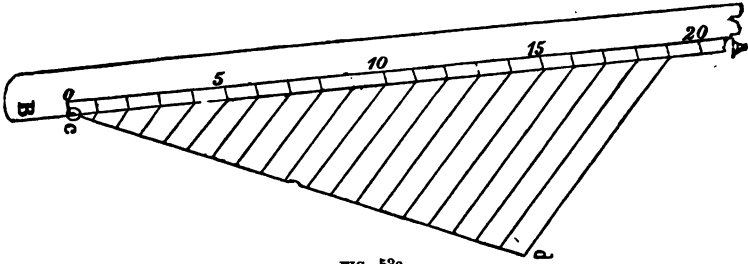


FIG. 53a.

point 19 of the scale A B, and draw, *parallel* to 19 d , lines through all the other points of the graduation; these lines will cut $c d$ into nineteen equal parts.

CHAPTER III.

VERTICAL SYSTEMS OF HILL-SHADING.

128. THE VERTICAL SYSTEMS of representing hills express the inclination of a hill-side by means of its *lines of greatest descent*, which may be described as the paths along which balls would roll, or water would flow, down the hill-side. Each of these lines is always normal (perpendicular) to the contours and indicates the *true direction* of the slope.

But, besides the direction of the slope, it is also necessary to show *the degree of its inclination*, and this is expressed by means entirely conventional (see Pl. II.). Several methods of shading have been adopted for this purpose.

129. LEHMANN'S SYSTEM.—This depends upon the principle of vertical illumination, in which the maximum light is reflected upwards to the eye by a horizontal surface, and a minimum by a surface inclined 45° to the horizon; the latter, being the steepest slope at which the earth will stand, is taken as the limit of least illumination. This is the English and German convention, and lays more stress on the different proportions of black to white in indicating the degree of slope than upon the distances between the shading lines.

When the surface of the ground is illuminated by vertical rays, it is evident that the level or horizontal parts will reflect upwards to the eye (which is supposed to be situated vertically above every point of the map) the greatest quantity of light; for the incident rays then coincide in direction with the reflected ones, the rays of light being supposed to be parallel. But when a vertical ray falls upon a surface inclined 45° to the horizon, the reflected ray will be horizontal, since both rays make angles of 45° with the line drawn perpendicular to the surface (*Fig. 21*).

130. *Lehmann's scale of shade*.—The natural limits of the declivity of slopes being 45° for the greatest, and a dead level for the least, it is required to apportion the illumination between

these limits, taking black to represent a slope of 45° , and white a horizontal plane. Divide the line A B (*Fig. 22*) into ten equal parts, corresponding to the ten steps of a gradation (by 5° at a step) from a level to a slope of 45° . Upon the line C D, below, represent in these divisions the different inclinations, viz., a level, a 5° slope, a 10° , 15° , etc., to 45° . The level is perfectly white. Then there are nine degrees of color to be determined, from 5° , which is the lightest shade, to 45° , which is black. Divide, now, each of the ten spaces of A B into nine equal parts. To exhibit the proportion of black to white in a slope of 5° , make one of those parts black; for 10° , make two black; three for 15° , and so on to 45° where all nine parts are black.

This scale of color shows the following proportions of black to white for all the inclinations (differing by 5°) from a level to a slope of 45° . All the nine parts of the part of the scale corresponding to the level are white. In a 5° slope, one part of the nine is black, showing a proportion of black to white of 1 to 8. In 10° , two parts out of the nine are black, showing a proportion of 2 : 7, or 1 : $3\frac{1}{2}$. For 15° , the proportion is 3 : 6, or 1 : 2. For 20° , 4 : 5, or 1 : $1\frac{1}{4}$. For 25° , 5 : 4, or $1\frac{1}{4}$: 1. For 30° , 6 : 3, or 2 : 1. For 35° , 7 : 2, or $3\frac{1}{2}$: 1. For 40° , 8 : 1. For 45° , all is black. Expressing these proportions in the form of ratios, we shall have the following table, in which the numerator signifies the quantity of black, and the denominator the quantity of white :—

<i>Level.</i>	<i>No.</i>	<i>Black.</i>	
5°	$\frac{1}{8}$	"	
10°	$\frac{2}{7}$	"	
15°	$\frac{3}{6}$	"	
20°	$\frac{4}{5}$	"	
25°	$\frac{5}{4}$	"	or $\frac{1}{4}$ more black than white.
30°	$\frac{6}{3}$	"	or $\frac{1}{2}$ " "
35°	$\frac{7}{2}$	"	or $3\frac{1}{2}$ " "
40°	$\frac{8}{1}$	"	or 8 " "
45°	all	"	

From the foregoing we deduce the two following practical rules.

131. *To find the ratio of black to white for any given slope : Subtract the given inclination from 45° for a denominator, and take the given inclination for a numerator, and we shall have the*

ratio as in the table in *Par.* 130. Apply this, for example, to the expression of a 20° slope. Take $45^\circ - 20^\circ = 25$ for a denominator. The numerator is 20; hence $\frac{20}{25}$, or $\frac{4}{5}$, is the ratio of which the numerator represents the black (see *Par.* 130).

132. *To find the inclination*, the ratio of the black and white being given, or having been observed from a drawing: *Multiply the numerator of the given ratio by 45° for a new numerator, and add together its numerator and denominator for a new denominator. Reduce the fraction.* For example, in reading a drawing, we find in a certain part a ratio of black and white expressed by the fraction $\frac{4}{5}$. Then

$$\frac{5 \times 45}{5 + 4} = \frac{225}{9} = 25, \text{—the inclination required.}$$

133. *To reduce these ratios to practice* in the drawing of shading lines, the four horizontal strips below the lines E F, G H, I K, and L M (*Fig.* 22), show how the black portion of each set of nine parts is divided up into lines of proper thickness for use. For example, in the strip E G of the rectangle *or sp*, which corresponds to the slope of 5° , the single black part above is subdivided into two black lines of half its thickness showing a proportion of 2 : 16. Each of these lines is, in the strip G I, divided into two of half the thickness of those in E G, making four shading lines, showing a proportion of 4 : 32. Dividing these in the same manner, we obtain, in the strip I L, a proportion of 8 : 64, and in the next strip below, of 16 : 128.

In the preceding calculations, as well as in the scale of shade, no variations of less than 5° have been regarded; but smaller differences may be used, if desirable, and the scale drawn, and the ratios calculated, in the same manner.

134. *To apply the scale to a drawing*, the scale must be cut off along the line L M. The part L C D M will then furnish us, along the line L M, with a graduated edge by means of which the distances between the centres of the shading lines can be marked off, and the line C D will show the different slopes to which the graduation corresponds.

Having, now, the horizontal sections of a hill given, write upon it with a pencil, in as many places as is necessary, the degree of the inclination, and bring the line L M of the scale tangent to the upper curve of the zone at that part of the scale correspond-

ing to the inclination we are required to express, and mark off from its edge the distances between the centres of the shading lines. Through each of the points thus determined, draw a line of greatest descent, copying from the scale for each group of the lines, the exact proportion of black to white. The shading in each zone will then express both the degree of the slope and its direction. These shading lines are called *hachures*.

135. *Working scale*.—A very convenient form of working scale is illustrated in *Fig. 30*, Pl. III. It consists merely of a rectangular strip of drawing paper marked on one edge with a scale of contours corresponding to any particular scale of construction (in the figure, $\frac{1}{8}$ inch), and having on one of the other edges a suitable scale of shade, both scales being numbered as indicated in the figure. The contour scale, being applied to the drawing, gives, approximately, the degrees of inclination, and the spacing and thickness of the hachures is then taken from the scale of shade.

The number of strokes to one inch and, therefore, the fineness of the lines, should, in some degree, comport with the scale of construction. In fine drawing, a spacing corresponding to 50 strokes to one inch is generally employed, though a skilful draftsman may be able to apply 75 strokes per inch. For the larger scales of construction, 40, 30, and even 20 hachures to the inch may be used.

136. Although the representation of the ground is thus confined to geometrical rules at all points, it must not be thought necessary to repeat the process of construction for every line. It will suffice to do so at those places where the slope exhibits the greatest variations. Thus, a group in each zone will be constructed where the slope is least, and again, where it is greatest, then a few intermediate ones. By graduating the changes in the shading lines, in passing from group to group, both as to their thickness and their intervals, we can easily fill the vacancies between the determinate groups. Without being mathematically exact, we shall thus obtain a result sufficient for all practical purposes, and as accurate as can be reasonably looked for in employing the lines of greatest descent.

137. In regard to the lengths of the shading lines, no absolute rule needs to be laid down. As there is no contemplated relation between the declivity and the length of the shading line,

this part of the work is left to the skill and discretion of the draftsman. It may be remarked, however, that if we confine their lengths to the width of the zone, their thickness in a slope of 45° would sometimes exceed their length; and in a slope of 5° , the lines would often be too long for convenience. In the latter case, this difficulty is obviated by dividing the zone; and in the former, the lines may be drawn across 2 or 3 zones, according to the scale, and other circumstances. The extremes of length for ordinary scales may be set down at $\frac{1}{8}$ of an inch for the steepest slopes, and about $\frac{1}{4}$ of an inch for the gentlest.

138. *General directions for pupils.*—In representing declivities by this method, considerable practice is required. This should be commenced by drawing repeatedly the scale of shade, until the eye and hand have become somewhat accustomed to the spacing and thickness of the hachures; after which, the pupil may proceed to apply the scale to the representation of slopes. *Figs. 31, and 33 to 39, Pl. IV.*, have been prepared with the view of aiding the learner to advance gradually from the easier problems to the more difficult ones in hill shading.

All the preparatory pencil lines having been drawn lightly, and the spaces for the shading lines laid off by dots, begin the shading at the steepest part of the upper zone of the hill. Draw the shading lines firmly, from curve to curve, so that each row of lines may terminate evenly at the lower edge; and introduce auxiliary curves wherever it is necessary to make the hachures shorter than the distance between contours, in order to preserve the shade (see *Fig. 39*). Draw always toward the body, turning the drawing on the table for that purpose. Draw these shading lines always *down the slope*, and proceed with them from left to right, so that the line just drawn may be uncovered by the pen and the distance to the next one be seen. Go all around the upper zone in this way, finishing by joining the row at the point of setting out. This is always effected more easily and neatly in the steepest part of the slope. After finishing the first zone, proceed to the second, and so on to the foot of the slope.

Where the curves are nearly parallel, the shading lines are straight and, also, nearly parallel; but when the curves depart much from each other, the shading lines, being normal to the curves, will themselves have some curvature, in order that they may tend perpendicularly upon a second curve which is not par-

allel to the first (*Fig. 31*). In such cases it is necessary, besides drawing the medial auxiliary curves, to put in lightly with a pencil, at short distances, a number of normals which, being carefully studied, will tend to correct or confirm the directions of the shading lines as they are drawn. In introducing additional curves in those parts of the hill where the slope is slight, care must be taken not to shorten too much the length of the shading line, for where the interval between them is large, the line must be proportionately finer and longer.

Any change in the direction, thickness, or proximity of the shading lines required by the different inclinations, must be effected gradually, and all sudden changes in those particulars must be carefully avoided. Any two consecutive lines of a hill should be approximately equal, similar, and parallel. Also, any change in the color or expression of contiguous zones must be made gradually. If it be required to pass from a light zone to a dark one below it, make the lower extremities of the lines of the light zone a little heavier, so that they may meet the upper-extremities of the lower row with nearly the same color. The latter may also be lightened a little. As zones differ in inclination, so, of course, will the spaces between the shading lines vary. No attempt, therefore, ought to be made to prolong the lines of one zone into the zone below. The lines in each row are manifestly independent of each other in that respect. It is only necessary, as above stated, to avoid sudden changes of color in passing from zone to zone. Even in a perfectly uniform slope, it will not do to prolong the lines thus, because it gives a hard and bad character to the style. But in the case of a conical hill, as in *Fig. 33*, it would give rise to an error in principle; for, soon after leaving the summit, we should have too few lines of descent to cover the ground, and they would be so far separated as to lose their connection, causing great meagreness of style.

The same rules for joining the different rows of lines are to be observed here, as in the Horizontal System, viz., the extremities of one set of lines must not protrude within a neighboring set (*Fig. 32, B*), nor must a vacancy be left, as at *A*. The rows must be accurately joined, without showing either a white line at their junction, as in the latter case, or a dark one, as in the former. The method of joining them is shown on a large scale at *C*, where the lines of the lower row falling between those of

the upper, start from a line which connects the lower extremities of the upper row. When the whole plan of the hill has thus been covered with lines of greatest descent, the base and summit must be softened off, by tapering to a fine point the lower end of each line at the base, and the upper end at the summit, turning the drawing upside down, if necessary, to accomplish this more easily. Or, these tapering lines may be drawn in beginning and ending the hill. The hill being finished, the pencilled lines may be removed. The directions here given for executing the drawing of a hill, will apply to a hollow, the shading lines of which are converging (see *Figs. 35 and 39*).

139. *Clay banks, steep ravines, and rocks.* These are always exceptions to the law of slopes, and, save in the case of rocks, cannot be regarded as permanent forms of the ground, since they are constantly undergoing reduction by the action of natural causes; therefore, the method generally adopted to show them is to make their shading lines exceptional also. In both Systems, this is done, in earth slopes, by shading them with a set of short and very black lines drawn perpendicular to the horizontal sections, or contours (*Figs. 12, 34, 35, &c.*; also Plate VI.).

140. *Ravines* with abrupt edges are usually represented as in the left hand portion of *Fig. 34*; while the representation of those with more gradual slopes is indicated on the right of the same figure, and in other figures.

141. *Cliffs, or faces of rock*, are shown in *Figs. 35 and 38*, and in Plates V. and VI. *Rocky ledges* (same plates) are represented with less definite outlines by coarse, wavy strokes, irregularly broken, drawn at right angles to the hachures.

These signs should be applied to the map before the hachures are drawn; and in drawing the latter, it is sometimes desirable to leave an occasional white space between the upper edge of the rock face and the hachures immediately above it.

142. THE SCALE OF SHADE OF THE U. S. COAST SURVEY.—The basis of this may be said to be Lehmann's scale. In applying the latter to the maps of the Coast Survey, it was found to be wanting in range; it was also thought that its expressiveness could be increased by modifications in the upper and lower ranges of the scale. After careful tests, a scale was finally adopted which, while essentially agreeing with Lehmann's scale from 5° to 25° , diverges therefrom by following a curve of natural sines from 25°

to 40° , and is enlarged by providing for the representation of inclinations from 40° to 75° , and from 5° to 1° , according to the following arbitrary rules. Retaining the width of the white space corresponding to 40° unchanged for all steeper slopes, at 45° , the distance between the middles of hachures is increased 25 per cent. above that at 40° ; at 55° , the distance is 50 per cent. greater; at 65° , the distance is 75 per cent., and at 75° , 100 per cent. greater than for 40° . Similarly, retaining the width of the black line for 5° unchanged, the spacing for 4° is made 25 per cent. wider than for 5° ; for 3° , 50 per cent. wider; for 2° , 75 per cent.; and for 1° , 100 per cent. wider than that for 5° .

143. The following is a table showing the proportions of the black and white spaces and the number of hachures to one inch for a scale of construction of $\frac{1}{80000}$.*

SLOPE.	PROPORTION OF BLACK TO WHITE.	NO. OF HACHURES PER INCH.
1°	1 : 21	50
2°	1 : 18	57.1
3°	1 : $15\frac{1}{2}$	66 $\frac{2}{3}$
4°	1 : $12\frac{3}{4}$	80
5°	1 : 8	100
10°	1 : $3\frac{1}{2}$	100
15°	1 : 2	100
20°	4 : 5	100
25°	5 : 4	100
30°	3 : 2	100
35°	7 : 3	100
40°	4 : 1	100
45°	$5\frac{1}{4}$: 1	80
55°	$6\frac{1}{2}$: 1	66 $\frac{2}{3}$
65°	$7\frac{3}{4}$: 1	57.1
75°	9 : 1	50

Standard
number of
strokes per
inch.

* Coast Survey Report for 1860, Appendix No. 20, paper by E. Hergeshelmer.

100 strokes per inch is here taken as the standard.

144. For other scales of construction, the standard number of strokes per inch used on the maps of the Coast Survey is as follows :

Scale of	100 strokes,	40	strokes to one inch
" "	200 strokes,	50	" " "
" "	300 strokes,	65	" " "
" "	400 strokes,	80	" " "

145. Another modification of the scale of shade has been very generally adopted in England, which, for ordinary purposes, has the advantage of simplicity and facility of application. It consists in establishing with accuracy the proportional quantities of black and white for three medium slopes only, viz., 15° , $22\frac{1}{2}^\circ$, and 30° , a level being represented by white, and a slope of 45° by black. Thus, by *Pars.* 131 and 132, we have this table of slopes with their proportions of illumination :—

Slope.	Proportion of	
	Black.	White.
level.	—	all
15°	1	2
$22\frac{1}{2}^\circ$	1	1
30°	2	1
45°	all	—

A scale of shade may be constructed from this table, by assuming the thickness of the shading line for the medium slope of $22\frac{1}{2}^\circ$. Having drawn this portion of the scale (as at B, *Fig.* 28), with equal proportions of black and white, diminish by one-third the thickness of the black line for the part A of the scale, and increase its thickness by one-third for the portion C. These divisions will represent slopes of 15° and 30° respectively, white and black being the extremes. All intermediate inclinations are, of course, indicated by graduating the thickness of the shading line by the eye.

146. The scale of shade may evidently be still further simplified by being reduced to a mere graduation by the eye, varying uniformly from the heaviest lines which it is designed to use on the drawing down to the lightest.

147. THE FRENCH SYSTEM OF HILL-SHADING depends more upon the distance between the hachures than upon the shade produced, although, in it also, the shading is graduated from dark to light, according to the declivity to be shown. In this method, slopes are expressed by a fraction the numerator of which is unity, and generally stands for the vertical distance between the horizontal sections; while the denominator is the

horizontal distance between them, expressed in terms of the vertical distance as a unit. For example, let $a b$ (*Fig. 23*) be the profile of a slope cut by horizontal planes at a and b ; call $b c$, 1 (unity), and designate the line $a c$, by s . Then the slope will be represented by $\frac{1}{s}$. If $b c$ is contained three times in $a c$, then the expression for the slope will be $\frac{1}{3}$. The limits between which slopes are represented in this method are, $\frac{1}{4}$, or 45° , for the greatest, and $\frac{1}{84}$, or $0^\circ 53' 43''$, for the least,—all slopes less than the latter being regarded as levels. The largest scale that will admit of conveniently drawing the lines of greatest descent is $\frac{1}{84}$ of the full size, or 6 inches to 100 yards, being about $8\frac{1}{2}$ feet to a mile. In drawings made to this scale and smaller ones, the vertical distance between the horizontal sections is generally one yard. For the scale of $\frac{1}{84}$, we shall have, as the least width of zone, $\frac{1}{84}$ of an inch, and for the greatest,

$$\frac{6}{100} \times 64 = 3\frac{84}{100} \text{ inches.}$$

To fill a zone wider than this with lines of descent, would be inconvenient and unnecessary.

148. *In order to determine the interval between the shading lines for any given inclination (which interval is always reckoned from centre to centre of the lines), we have the following rules:—*

Rule 1. To the distance (measured along the line of greatest descent) between the upper and lower curves of any zone, add three-tenths of an inch; a sixteenth part of this sum will be the proper interval for the shading lines. For example, if the given inclination is $\frac{1}{84}$, the scale being $\frac{1}{84}$, and the zones 1 yard thick, the width of zone for $\frac{1}{84}$ will be $.06 \times 60 = 3.60$, or $3\frac{6}{10}$ inches; to this add $\frac{3}{10}$ of an inch, and divide by 16, and we will have 0.244. If the inclination is $\frac{1}{4}$, the interval is 0.0225 inches.

149. To save the labor of calculation, the following is a practical method of laying off these points by the eye. Cut a rectangle of paper, the side $A B$ of which (see *Fig. 24*) must be equal to $\frac{3}{16}$ of an inch. It is required to space off the shading lines at MN , which is a line of greatest descent. Apply the rectangle near the middle of MN , with the side AB laid in the direction of the middle of the zone, and towards the left. Prolong AB to the right, and make BE equal to MN . Then AE

will be equal to the width MN of the zone, plus $\frac{3}{16}$ of an inch—which distance is to be divided into 16 parts. Now, by the eye (or with compasses, until the eye is sufficiently practised) halve the line AE, then quarter it, and treat each of the quarters in the same manner; through each of the 17 points so found, a shading line is to be drawn across the zone.

150. When the curves are nearly parallel, we can easily, in this manner, determine seventeen lines at once. But when the curves depart from each other rapidly, the lines of greatest descent also diverge proportionately, and it becomes necessary to divide the line of greatest descent into four or more parts. The true line of slope GI (*Fig. 24*) is thus divided, and by similarly dividing others, as at YZ, etc., we can draw the auxiliary sections *ab*, *cd*, and *ef*. These sections being approximately parallel, we are now prepared for the application of

Rule 2. To a quarter of the distance (measured as before) between the upper and lower curves of any zone, add $\frac{1}{16}$ of an inch; a fourth part of the sum will be equal to four intervals.

Cut a rectangle of paper, as before, whose side A'B' shall be $\frac{1}{16}$ of an inch. Apply A'B' to the middle point of the normal drawn at F (*Fig. 24*). Make B'P equal to FK, and divide A'P into four parts, by halving and quartering as before. This will determine 5 lines, to be drawn as far as the auxiliary section *ab*. Apply A'B' to the second normal, and similarly determine four lines of the second auxiliary zone, and so on for the others.

151. *The thickness, or breadth, of the hachures is determined by assuming them to vary directly as the inclination; hence,*

Rule 3. For a slope of $\frac{1}{4}$ the thickness of the shading lines is equal to two-thirds of the distance between their centres, and their thickness will diminish with the inclination, down to $\frac{1}{4}$, where the lines will be as fine as they can be drawn. This rule will always, in a slope of $\frac{1}{4}$, make the shading lines twice the breadth of the white space contained between them (*Fig. 25*). Thus, for a scale of $\frac{1}{16}$, the zone being 3 feet in height, and the slope $\frac{1}{4}$, the intervals are 0.0225 inches, two-thirds of which is 0.015, or $\frac{1}{66}$ hundredths of an inch, nearly. In all cases, the shading lines decrease to the finest line, to express the slope of $\frac{1}{4}$.

152. For marking off the intervals, and drawing shading lines of the proper thickness, a movable tangent scale may be constructed as follows:—Draw a right line A B (*Fig. 27*). At A, draw

perpendicular to it, and downwards, a right line, whose thickness and length shall both correspond to a slope of $\frac{1}{4}$, according to the scale of the map, and lay off towards B, the first interval of the scale, which must correspond to a slope of $\frac{1}{4}$ (*Rule 1.*). Find the interval for $\frac{1}{8}$. Now, from $\frac{1}{4}$, the intervals increase uniformly up to $\frac{1}{8}$. They may, therefore, be regarded as an arithmetical series, whose first and last terms are respectively the first and last intervals, and whose common difference is the difference of the extremes divided by the number of intervals. Each successive interval can then be found by adding the common difference to the preceding interval; and finally, by successive additions, they can all be laid off from A. When the scale is small, the common difference may be doubled, and the intervals laid off by two at a time, and afterwards halved; or it may be quadrupled, and four spaces laid off at once, and then divided into four parts. Let fall, now, through the points thus determined, perpendiculars to AB, of indefinite length, and varying uniformly in thickness from the first line at A to the last one at B, which is as fine as possible. To determine the width of the zone to which the intervals and thickness of these lines correspond, start from the first line, already drawn of the proper length at A, and, from that line, increase the length of each perpendicular up to the 64th, in an arithmetical progression whose first term is the least width of zone and the last the greatest width, employing a similar method to that used for spacing. Draw through the lower extremities of these lines the curve CD, and the tangent scale will be completed. Each division of the line AB is the interval between two shading lines, corresponding to a width of zone equal to the mean of the two perpendiculars which limit that particular division.

To make use of this scale it must be cut out, following its outline, A B C D A, and applied to the contours (*ef* and *gh*, for example), as shown in *Fig. 27*. The degree of the inclination, the spacing, and the corresponding thickness of the lines, can then be read off, as at *m*, *n*, *o*, *p*.

153. Upon every drawing made in accordance with the above rules, there should be placed a scale which will conveniently show the relation between the width of the spaces and the slope of the ground at any part of the map.

The following is a convenient form for this scale:—Divide a line AB (*Fig. 26*) into 64 equal parts, numbered from zero to

64. Let fall perpendiculars from A and B. On that at A, measure downwards to *a*, a distance equal to the width of four spaces corresponding to a slope of $\frac{1}{4}$, and at B measure downwards to *b*, a distance equal to four spaces, expressing a slope of $\frac{1}{4}$. Join *a* and *b* by a right line. Let fall, now, perpendiculars from every point of division until they intersect the line *ab*, and the scale is completed.

To make use of it in finding the declivity expressed by the shading lines, take off in the dividers the width of four spaces at any part of the drawing, and compare it with the lengths of the perpendiculars to AB. If, for example, it is found that it coincides in length with No. 12 of the scale, then the slope expressed by that interval is $\frac{1}{4}$.

Reciprocally, this scale may be used to determine the width of four spaces.

154. OLSEN'S SYSTEM. *—This system, applied to the topographical map of Denmark, is essentially a horizontal system supplemented, in slopes steeper than 15° , by flat shading with hachures, the depth of shade varying according to the steepness of the slope. A peculiarity of the system is that, as the steepness increases, the height of each zone is doubled at certain points in the scale of inclinations by omitting alternate contours. Thus, from 5° to about 15° , the zones are 5 feet high; from 15° to 25° , they are 10 feet high; from 25° to 35° , they are 20 feet high; and from 35° to 45° , they are 40 feet high. The five-foot zones are left white, but the others are shaded with equidistant hachures whose thickness varies somewhat in proportion to the heights of the zones. Inclinations of 45° and upwards are made black. It would be better, however, as suggested by Mr. Enthoffer, to extend the hatching with lines of appropriate thickness beyond 45° . The above statement of the different groups of inclinations, and of the extent of the distinctive shading in each group, is only approximate.

155. Although in all of the more important methods of employing lines of greatest descent, just explained, the theory of the convention is expressed by fixed rules, yet it is admitted that in practice an approximation only is obtained; inasmuch as a great deal is left to the judgment of the draftsman's eye, and his skill of hand. Still, it is not only useful, but necessary, to have some

* So named in Enthoffer's manual.

fixed principles as a basis on which the art may be founded, and to which practice may be conformed. The student ought, therefore, to make himself perfectly acquainted with the method (whichever it be) that he adopts, so as to present throughout his drawings a consistency in the expression of at least the relative degrees of inclination. For popular use, on ordinary occasions, this will be sufficient ; as almost every one who is interested in looking at a topographical map will have learned enough of the conventional signs to comprehend their general intention ; while the scale of shade or of spaces, which should always be put upon a drawing, will inform those who are disposed for a minuter reading of it if it is geometrically correct.

CHAPTER IV.

COLORED TOPOGRAPHICAL DRAWING.

156. *Preliminary steps.* As has already been intimated, water-color drawings can be made properly only on either stretched or mounted paper of suitable quality. In employing freshly mounted sheets for this purpose, it is always best to leave them adhering to the board until the drawing has been finished ; otherwise the paper is apt to become slightly distorted by the application of broad tints.

Having prepared the paper, the lines of the drawing are put upon it, first in pencil, and afterwards with a very fine ink line. The ink, although black,* must not be thick ; for, after the lines (outlines only) of all the shores, roads, buildings, etc., have been inked, and all the pencilled lines erased, the drawing must be *washed*, either by exposing it to water dashed over it, or by quickly passing a soft sponge, well saturated, across its surface. This is done for the purpose of removing those portions of the ink which a wet brush would detach from the paper, and which, mixing with the tint, would injure its purity. When dried, the drawing will be ready for the coloring.

157. *Conventional tints*, see *Fig. 69* :—

Water, a flat tint of pure Indigo or Prussian Blue, light colored (a).

Sand, a flat tint of Yellow Ochre (b).

Cultivation, a flat tint of Burnt Sienna (c). This is sometimes ruled with brown, yellow, or green in a manner suggesting furrows *Fig. 69*, at (c). But these colors should be merely of the consistency of heavy tints.

Grass Land, or *cleared ground*, a flat tint of green, of Hooker's Green No. 2, or of a mixture of Prussian Blue and Gamboge (d).

* Sepia or Burnt Umber may be used instead of India Ink.

158. Only very clean water (distilled water is the best) should be used in mixing colors. Some of the colors, as Prussian Blue and Gamboge, give a transparent tint on being mixed, while others are heavier and yield a large amount of sediment. The best way to prepare these granular colors for tinting is to precipitate the particles by allowing the mixture to remain undisturbed for a few minutes, whereupon the clear tint can be carefully poured off into another saucer and used. The mixture should be of a deeper color than that which it is purposed using, and the proper intensity may then be obtained by diluting the clear tint with water.

159. *Tinting*.—In laying on a flat tint, particularly upon soft or porous paper, first wet the surface to be colored with clean water, and after the water has been absorbed (the surplus can be removed with a piece of clean blotting paper, which should always be near the colorist), and while the paper is yet damp, incline the lower edge of the drawing towards the body, and begin applying the tint with a full brush at the top of the figure. Draw the brush carefully along the upper edge, and for a short distance downward along the right and left hand edges; then sweep from right to left and from left to right, alternately, across the intervening space, thus bringing the tint down in horizontal bands or stripes; these must not appear, however, after the tinting is finished. Work very carefully near the edges, using the point of the brush more than in the interior, where the movements must be more rapid, in order to prevent the color from setting or drying on the lower edges; and replenish the brush with fresh tint, whenever necessary, in order to keep the surface of the paper well wetted. On arriving near the lower outline of the figure, the quantity of tint must be diminished, so as to leave just enough in the brush to finish without allowing the color to accumulate upon the lower outline. In no case, anywhere on the drawing, must the color be allowed to lie in puddles, or drops. When the colored water has once thus flowed over the paper, the tint is finished, and must not be touched again; for if there be any little defect in it, one trial will show that any attempt to remedy it while the color is drying will only make it worse. As the hand acquires skill, it will generally be found that tints are better in proportion as they are more quickly laid on.

160. Should stains or patches occur, they may be remedied by wetting the whole drawing and gently washing the faulty parts with a brush or soft sponge, and repeating the tint lightly, should it be too much reduced by washing. Tints that are too strong may be rendered weaker by sponging, as above described, and some may even be removed entirely. White spots left in a tint may be filled up, after it is dry, with the point of the brush, taking care not to apply the color where the paper is already tinted, as that would double its intensity, and make a dark spot. The knife should never be used for erasing on a tinted drawing, as the color sinks and becomes intense wherever the paper is scratched.

161. After the flat tint it will be necessary to practise the *alternate* or double tint. This consists of two colors, applied alternately, their edges being allowed to blend into each other. For this purpose, two saucers of tint must be prepared, with a brush for each. Begin with one of these colors at the upper outline of the figure, as for a flat tint; having laid on a small portion of that tint, take the brush charged with the other color, and before the first dries, run around its edge with the second, allowing them to blend together; then resume the first tint, blending in the same manner; continue the alternation throughout the space to be filled. It will be observed that the surface of the paper is here treated exactly as if a flat tint were being laid, the color not being allowed to dry anywhere. Each tint is spread upon the white paper, and therefore shows itself pure. The forms of the masses of each color should be varied, and not made in stripes, or spots, but should be irregularly clouded. The tints in this sign should be light, and equal to each other in strength.

162. *Uncultivated Land*, or *Brushwood*; is represented by alternate tints of Hooker's Green, as for cleared land, and Burnt Sienna, laid on as just described. This sign may also be made with alternate Green and Crimson Lake, it being the only double tint used (e).

163. *Structures of Masonry*, such as buildings, bridges, locks, walls, etc., are tinted with Crimson Lake, and shadowed with a neutral mixture of Indigo, Burnt Sienna, and a little Lake; while *frame buildings* and other structures of wood may be distinguished by a tint of yellow, relieved with a shadow as in the

preceding case. For this and all other purposes of light and shade, as in forest, etc., the light is supposed to enter the drawing at the upper left hand corner of the margin, in parallel rays inclined 45° to the horizon. The shadow of any object will therefore surround its lower and right hand outlines. After the shadow has been introduced, the outline of the object must be strengthened, by making it a heavy black line on the sides opposite the light (f). This is always the last work on the drawing, and must never be undertaken while any brushwork or tinting remains to be done.

164. *Roads, Streets*, and all portions of the drawing not particularly described, are tinted with Yellow Ochre.

165. In the signs for *Forest and Marsh*, some attempt is made at a resemblance to the things signified, and a more minute description of the method of executing them will be required. For *forest* (*Fig. 69, g*) the ground is first prepared by laying on a flat tint of green, exactly as for cleared land. Then with a very hard and sharp lead pencil, or a pen with pale ink, groups and masses of trees are drawn *in outline*. Then with a tint of Hooker's Green No. 2, a little more intense than the ground color already laid on, each tree and mass of foliage is shaded on the right hand and lower portion. An orange tint made of Gamboge and Burnt Sienna is then touched upon the side next to the light. These two tints should just fill up the outline, the green occupying about two-thirds of the figure. This finishes the tree. It only remains to add the shadow upon the ground, which is made and laid on as directed in *Par. 163*. For single trees, as in orchards, the shadow is detached, but in masses of foliage, it is laid close to the outline (h). The above is merely a formula for this sign. It may be departed from whenever, as in colored maps drawn to a large scale, a better effect may be produced by giving the trees a sketchy appearance.

166. In colored plans for ornamental planting, which are almost always drawn to a large scale, it is allowable to represent evergreens and, to a limited extent, single deciduous trees in partial elevation (*Fig. 71, Frontispiece*), the object being to facilitate the reading of the map by persons unacquainted with conventional symbols. For the same reason the representation of masses and groups of deciduous trees and bushes is not subject to any particular convention. Nor is it necessary that all the

trees should be shown strictly in plan ; the general effect is improved, rather than otherwise, by suggesting the sky-line of important groups by means of tree tops projecting, here and there, in elevation above the tree-shading around them. The proper mode of drawing trees in such plans resembles so closely the pencilling of trees in perspective sketches, that it may be safely said that a practical knowledge of free-hand drawing is an indispensable preliminary to this kind of mapping. The outlines and shading lines of the trees are drawn with a soft pencil, after which, the colors are applied, beginning with the lighter ones and ending with the darker shades. The shade lines may then need retouching by the addition of a stroke, here or there, with the pencil or with a finely pointed stiff brush, using a dark neutral mixture of green. But this must be done very circumspectly ; otherwise, the introduction of unnecessary lines will tempt the draftsman to introduce others in the hope of disguising previous errors, and may, thus, ultimately ruin the drawing.

The difficulty of representing trees in the manner described increases very rapidly with the diminution of the scale, and a practical limit is reached in a scale of 100 feet to 1 inch. The scales of such drawings average about 40 feet to 1 inch.

In colored maps drawn to a small scale, woodland should be represented in pencil, somewhat after the manner shown at *a*, Pl. I., *Fig. 8* ; though coarser, and not so dark as the sign referred to.

167. *Marsh*, as in pen-drawing, is composed of a mixture of the signs for water and cleared land which interlace horizontally, that is, always parallel to the base of the drawing (*i*). The green tint of marsh is first to be laid on with a brush moderately charged with the color for cleared land. In doing this, attention must be paid to that part left white, which latter must be rather less in quantity than the green, and must resemble it in form, projecting its horizontal points into the green, just as the green projects into the white. The outer limits of a marsh must be made up of an outline of projecting green points. The water is represented with very light Indigo or Prussian Blue, laid on in horizontal streaks of varied width, just filling up the white space without encroaching on the green. When that is done, a thin shading line must be drawn along the lower edge of the green. This is composed of Indigo and Burnt Sienna,

and must be confined within the limits of the green tint, and not allowed to touch the white. The introduction of a single tree, here and there, assists the good effect of this sign.

168. A very effective sign has sometimes been used to indicate a fresh-water marsh or a wet meadow. It consists in covering the ground color of green representing the meadow with pale green stripes, laid on with the point of a fine brush, always parallel to the lower border line of the plate. The general effect is somewhat similar to that of *Fig. 9*, Plate I., and is quite suggestive. The striping should be done with the free hand, holding the brush nearly vertically. This sign is all the more effective if the stripes are allowed to blend, here and there, in the interior, producing a darker shade than near the edges.

169. In painting the trees in forest, and shading the banks of marsh, etc., it is not necessary to mix tints in a saucer. It will be sufficient to rub the three colors, Indigo, Burnt Sienna, and Gamboge, side by side upon the same plate, and with a proper quantity of water in the brush, to mix them to the proper color and intensity, so that they may be applied without lying in drops upon the paper.

170. *Hill-sides*.—In representing slopes, or declivities, with colors, the same principle is followed that is observed in pen and ink drawings, viz., that the steepness of the slope is to be indicated by the darkness of the shading. The tint used to replace that produced by the pen-lines is composed of Indigo and Burnt Sienna, when the ground-work is green; but when used over sand or cultivation, a little Lake is added to the mixture in order to neutralize its greenish hue. This shade tint is always laid on after the ground is covered with an appropriate sign. No hard edges must be admitted; the following method being used to avoid them (k):—With clean water in the brush, moisten well the paper along the line of the crest of the slope, and, before it dries, begin to lay on the shade of the slope in the lower edge of the moistened part. Proceed down the hill with it, laying it on like a flat tint, until the lower limit of the slope is reached, when, while the shade tint is still moist, it must be softened, or blended, with a brush and clean water. If the slope be of great extent, its sides may be shaded in successive portions, provided no hard edges are left on the tint. A slope can seldom be finished by tinting it once, but usually requires repeated tinting and re-

touching, until the proper depth of shade is reached, and all the detailed variations of declivity are indicated by corresponding degrees of intensity. The joints, or edges, of superposed tints should be made at different places, at each application, otherwise they will be visible.

171. *Inclosures* are represented as follows: Hedges, by green dots, varied in size, with shadows. Masonry or brick walls, by a line ruled with red. Wooden fences, by lines either ruled or hand-drawn with a pen containing the neutral mixture of Indigo, Burnt Sienna, and Lake (n).

172. In reference to the general effect of colors in tinted topographical drawings, two things are to be considered, viz., the quality of the mixed tints, and the strength or intensity of color. Greens should not be of a cold quality, such as is produced by too much blue, but should be of a bright hue. As to intensity, everything should be subordinate to the condition of clearness. Next, the tints must be clean, transparent, and rather light, so as not to mask any of the details of the drawing. They must be of sufficient strength, however, to be readily distinguished from each other at once, and even a very little stronger than necessary so as to allow for fading.

All tints that are much extended ought to be balanced, that is, no one ought to obtrude itself upon the eye by its too great intensity. According to the terms of art, everything should be "in keeping," and spottiness avoided. Thus, forest, brushwood, and cultivation, ought to be kept about equal in strength. Cleared land, marsh, sand, and water, may be made of equal intensity, but all a little lighter than the other signs. Tints that are of small extent, may be a little exaggerated in intensity in order that they may be readily distinguished. Buildings, being objects of small extent, and having a certain importance, require a well-marked red tint, shadow, and shaded outline. Villages, with their gardens, orchards, etc., should generally be represented a little stronger in the tints that compose them, than the general tone of the surrounding country (*Fig. 69 at m*).

173. The order in which the tints for the different signs are successively laid on, is a matter for the experience and skill of the draftsman to decide. It is generally thought better, in order to insure a proper balance among them, to begin with the lightest.

174. Many of these signs, together with others which will be readily understood without explanation, are combined in *Fig. 70* (Frontispiece).

Fig. 71 is intended to illustrate the use of colors in representing the ornamental arrangement of grounds. The purple areas striped with blue in the upper left hand corner of *Fig. 71*, indicate the location of the kitchen garden. Immediately in front of one of these areas, and separated from the clothes yard by a screen of tall bushes, is the green-house. The other features, it is believed, will be understood without further description.

CHAPTER V.

LETTERING, TITLE, ETC.

175. Next in importance to the proper delineation of the topographical features heretofore described, is the *lettering* of the map. In this department the draftsman's responsibility as to the good appearance of his drawing is very considerable, for nothing so surely detracts from the value of a map, viewed as a work of art, as an awkward and unhandsome style of lettering.

To make manuscript letters, imitating type-printing, requires a great deal of study and practice; and to proportion the title and other lettering, so as to suit the scale and general effect of a map, is a matter of considerable importance. As far as written directions can be of service in guiding the draftsman in this part of his work, the following rules may assist him.

176. 1st. As to the *time* for lettering. If the map is a *pen-drawing*, all letters that fall upon the surfaces of lakes, rivers, etc., or upon the sides of steep hills, or in a forest, should be put upon the drawing before those features are drawn, for it is easier to pass their characteristic lines over the letters than to draw the letters upon the paper already so occupied. In a *tinted drawing*, the letters are always the last drawn, as a brush cannot be passed over them without blotting.

177. 2d. As to the *size* of the letters. This depends upon two things; the importance of the object described, and the scale of the drawing. The different characters or types of lettering employed, are thus arranged in regard to importance: 1st, the upright CAPITAL; 2d, the inclined *CAPITAL*; 3d, the upright Roman, or ordinary small type; and 4th, the *small Italic*, or *stump print* (see Plates XIII, XIV). The capital letters belong to such objects as large cities, an extensive forest, a bay or gulf, an island, or a large mountain or river. These same objects, when they are of less importance, may be described in smaller

type, as may also a village. On the Coast Survey, the upright letters are used for land objects,—islands, points, etc.; the inclined letters for water,—bays, coves, rivers, etc.

178. For proportioning the height of the letters to the scale of the drawing, the following table is offered merely as a general suggestion, which is all that can be claimed for it, inasmuch as the size of the letters must, to some extent, be governed also by the size of the map; large letters would obviously be out of proportion on a small map.

SCALE.		HEIGHT OF LARGEST UPRIGHT CAPITALS.	HEIGHT OF SMALL LETTERS FOR EXPLANATORY NOTES, ETC.		
$\frac{1}{50}$ in.	or 1 in. to 50 feet	Six-tenths of an in.	Twelve-hundredths of an in.		
$\frac{1}{16}$ in.	or 2 ft. to 1 mile	Five “ “	Ten “ “		
$\frac{1}{32}$ in.	or 1 ft. to 1 mile	Four “ “	Eight “ “		
$\frac{1}{10}$ in.	or 4 in. to 1 mile	Three “ “	Six “ “		

The variation in height for each scale, from that of the upright capital to the italic, is gradual, and is regulated, as above stated, by the importance of the object.

179. The *thickness* of the capitals, in proportion to the size, is *one-seventh* of the height of the letter, though *one-sixth* makes a sturdier letter (see Plates XII, XIII, XIV).

180. It is desirable that all lettering should be in lines parallel to the base. Yet bays, rivers, roads, etc., sometimes require the lines of the letters to be both oblique and curved. In such cases, to facilitate the reading of names without requiring much change in the position of the drawing, the following rule should be observed:—Lines of lettering which are inclined toward the upper right-hand corner of the map should be printed so as to be viewed naturally from the right side of the map, while lines of lettering inclining toward the upper left-hand corner should be viewed from the left side;—in other words, the former should read from the bottom upward, the latter, from the top downward.

181. The *formation* of the letters requires great attention and study. The beginner must copy from good models all the different kinds of character, and thus acquire a perfect knowledge of the proper proportions and expression of every letter, both

large and small. He must then exercise himself in drawing them without a model, in order to acquire manual skill, which he cannot be said to have done until he is able to form all the letters correctly and give them their proper relative proportions and spacings, having for a guide only the two pencil lines which limit their height. In making capitals, each letter must be accurately outlined in pencil; the outline must then be drawn in ink, with a firm and steady line, and afterwards filled in with the pen. Whether capitals are upright or inclined, it is well to draw a few lines in pencil, parallel to the direction of the letters, which will serve as guides in drawing them.

182. The following table is offered as an approximate guide to the pupil in forming capital letters.

Taking the extreme width of H measured across the middle, or, exclusive of the caps, as the unit, the widths of the other letters or of their characteristic parts may be expressed approximately by the numbers in the third column of the table. In the case of letters having oblique lines, these widths are to be taken at the intersections of the outer oblique lines with the upper or lower limit of each letter. The caps are in all cases excluded.

LETTER.	PART MEASURED.	WIDTH, H BEING UNITY.	LETTER.	PART MEASURED.	WIDTH, H BEING UNITY.
A	bottom.	$1\frac{1}{15}$	N	—	$\frac{7}{8}$
B	{ top.	$\frac{1}{8}$	O	middle.	$1\frac{1}{4}$
	{ bottom.	1	P	top.	$\frac{5}{8}$
C	{ top.	1	Q	middle.	$1\frac{1}{4}$
	{ bottom.	$1\frac{1}{15}$	R	{ top.	$\frac{1}{8}$
D	{ middle.	$1\frac{1}{15}$		{ bottom.†	$\frac{1}{8}$
	{ top.	$\frac{1}{8}$	S	{ top.	$\frac{1}{8}$
E	{ middle.	$\frac{1}{8}$		{ bottom.	1
	{ bottom.	1	T	top.	$1\frac{1}{15}$
F	{ top.	$\frac{1}{8}$	U	—	$\frac{7}{8}$
	{ middle.	$\frac{1}{8}$	V	—	$\frac{5}{8}$
G*	{ top.	1	W	top.	$1\frac{1}{2}$
	{ bottom.	$1\frac{1}{15}$	X	{ top.	$\frac{5}{8}$
J	bottom.	$\frac{1}{4}$		{ bottom.	$1\frac{1}{2}$
K	{ top.	$\frac{1}{8}$	Y	top.	$\frac{1}{8}$
	{ bottom.	$1\frac{1}{15}$		{ top.	$\frac{1}{8}$
L	bottom.	1	Z	{ bottom.	1
M	—	$1\frac{3}{4}$			

* In all rounded figures the measure is from the vertical tangent of the extreme part of the round.

† The measurement is from the vertical part of the curve.

The horizontal bars of H, E, and F are at the middle of each letter; those of B, P, and R are very slightly above it; while the horizontal bar of A is from $\frac{1}{4}$ to $\frac{1}{2}$ the height of the letter below the middle. The outer curves of B, P, R are ovals, with the longer axes horizontal, while the inner curves of all letters are ovals having their longer axes vertical.

In ordinary type, the extreme width of the letter H, exclusive of the caps, is about $\frac{1}{4}$ of its height.

A general rule for *spacing letters* is to make the white areas between them approximately equal. But this rule cannot be strictly adhered to, on account of the peculiar forms of some of the letters.

183. In drawing the small roman and italic letters, the same kind of study with the pencil is at first required; but as the heavy parts of these are made at once, by a bold pressure upon the pen, the operation of making them resembles careful writing. As a preparation, three pencil lines are drawn, the lower two to form the upper and lower limits of the ordinary letters, and the upper one to limit the capitals and the tops of the l's, d's, etc. The parts of these letters are of two kinds, viz.: *curved* and *straight*, which should be carefully distinguished from each other. For example, a, c, g, o, s, etc., are composed entirely of curves. They must be symmetrically drawn, and the width of the ordinary letter must be only a little less than its height. The round part of g does not reach quite to the lower line. The letters b, d, f, h, m, n, p, q, etc., are composed both of curved and straight parts. The uprights of these letters must be made perfectly straight from top to bottom, with a little horizontal return, pointing to the left at the top, and at the bottom to the right. The m and n, although curved at the top, must be brought down straight to the lower line, with a return pointing to the right. These returns must always form a sharp angle with the line of the letter, and not be rounded. The letters i, k, l, v, w, x, and z, being composed entirely of right lines, care must be taken to keep their elements straight. See Plates XII, XIII, XIV, for the application of these rules. In these Plates, the small letters, both upright and slanting, are $\frac{1}{3}$ of the height of the capital letter beginning the word or line, and the inclination of all slanting letters is in the ratio of 3 parts of base to 8 of height.

184. The beauty of lettering depends more upon the regularity and the uniformity of style of the letters than upon their individual character. In italics, the inclination of the lines must be everywhere the same. (See Plate XIV.) As compared with clear roman or italic type, manuscript lettering ought to occupy rather more horizontal space, and always looks better when slightly extended; crowding it injures the effect very much. There is a peculiar beauty in lettering when executed properly with the pen, which does not depend upon a resemblance to printed work. Finally, the merit of a map even as to accuracy is not safe from doubt, when, however correctly drawn, its style of lettering marks a want of knowledge or skill in so simple a matter as the formation of the letters of one's own language.

185. A map having been drawn and lettered, there are required to complete it a descriptive title, a border, the meridian line, and the scales of distances.

186. First. *The Title.* This may be placed outside the border, if it takes up only one line; but if it requires several, then it must be placed within it. The letters composing the name of the *locality*, which is generally the most important word, should not exceed in height three hundredths of the length of the short side of the border. The letters of other words are varied in size according to the importance of the words they compose. The execution of the title furnishes another opportunity to enhance, by its ornamental character, the beauty of the drawing. It should occupy one of the corners of the map, and have the middle points of all its lines of words or phrases upon a vertical line. It should state, in small letters, the name of the draftsman, the dates of the surveys and of the drawing, and under whose direction executed.

187. *Arranging the title.* As a preliminary to drawing the title, first determine the relative prominence which should be given to its different words. The order of prominence is usually indicated by answers to the queries: 1°. What does the map represent? 2°. Where is the locality? 3°. For whom have the survey and map been made? 4°. By whom, and when?

Having decided this point, write out the words of the title in a round hand, arranging them in several lines, and using different

sized letters, according to the degree of importance of the words, and endeavor to balance the upper and lower portions with respect to the heaviest line, which is usually placed near the middle, remembering that such phrases as, "of the," "belonging to," etc., are always subordinate to the words which they connect, and that they can, therefore, be inserted, if necessary, between more important lines. This will usually suggest the outline of the title and the heights of the letters; also, which of them should be **CONDENSED**, and which **EXTENDED**.

Having done this, draw upon another piece of paper a vertical line, to represent the centre line of the title, and at right angles to it, other lines, properly spaced, to indicate the upper and lower borders of each line of letters. Pencil the outlines of the letters very carefully between these lines, working both ways from the centre in order to preserve the symmetry of the title, and complete all but the lines of very small letters, the general effect of which may be indicated by a wavy line or in some other suitable way. Alterations in the sizes of the letters or in the spacing, which now suggest themselves should be carefully made until the appearance of the title is satisfactory. The centre line of the title and the lines perpendicular to it are then drawn upon the map, and the lettering is transferred by means of tracing paper and carefully inked.

188. If a series of sectional maps of a region or of a line of communication are to be made to the same scale, their titles will have many words in common. In such cases, the first tracing should be preserved and used for reproducing on each map the recurrent words. After considerable experience, the draftsman may, in ordinary cases, dispense with the preliminary drafting, and may pencil each line of lettering separately on a piece of tracing paper, from which, after finding the centre, the letters may be transferred to the finished drawing; or else, the lettering of each line may be sketched out on a separate slip of paper, the lower edge of which being applied to the horizontal line on which the words are to be arranged will mark the proper places for the letters. The draftsman can then proceed to pencil and ink them. In the case of elaborate titles, however, it is safer first to arrange everything satisfactorily on a preliminary drawing.

189. Second. *The Border.* The taste and fancy of the drafts-

man may sometimes suggest such a composition of lines or figures for this purpose as will greatly embellish a drawing ; but if a plain border is required—and such an one is usually preferable—the style generally adopted is a double line a heavy one on the exterior, and a light interior one, the heavy line having nearly the same breadth as that of the blank space between it and the light line. As the map is generally a rectangle, the rule usually followed for proportioning the breadth of the border (including the two lines and the space between them) is to make it not more than the one hundredth part of the length of the short side of the rectangle.

190. Third. *The Meridians.* The true meridian is an indispensable adjunct to every finished topographical drawing or map. The importance of this line is evident, as without it no just idea of the situation of the locality with reference to the surrounding country can be obtained ; nor could the drawing be compared or used in connection with any other map, unless it has this fixed line of direction, which is common to all.

It is usual, therefore, to make this meridian line conspicuous, and to ornament its northern extremity with some fanciful device, although a simple arrow-head or a star, with the letter N, will answer the purpose.

The direction of the magnetic meridian at the time of the survey should also be indicated. This is generally done by drawing a less conspicuous line than that used for the true meridian through the middle point of the latter, and at an angle with it equal to the declination of the magnetic needle. The declination is inscribed either in the space between the lines or on the magnetic meridian. When both meridians are thus represented, it is customary to draw a full arrow-head on the true meridian, and to distinguish the magnetic North by half a head, drawn on the right hand or on the left, according as the declination is East or West.

191. Fourth. *The Scales.* Every map of territory in the United States should have at least two scales of distances drawn on it, in some convenient place near the lower border ; the one corresponding to the British units ; the other, to the metric system of linear measures. They should be carefully constructed, one underneath the other, so that their parts can be easily compared, and they should be appropriately numbered and lettered. For

convenience of reference their lengths ought not to be less than one-fourth of the longest side of the border.

The scale of the map should also be indicated in the form of a fraction, neatly printed below the title, the purpose being to facilitate the correct comparison of the scales of different maps at a glance.

CHAPTER VI.

FIELD SKETCHING.

192. *Field Books.* Field sketches are made with the lead pencil, and are drawn upon every page of the field book, or upon alternate pages, according to the kind of book used. The symbols employed are those given in Part II, Chapters I, II and III. In so-called *compass books* and *transit books*, the left-hand pages are ruled for the record of the instrumental work, and the right-hand page is reserved for topographical sketches; while *topographer's sketch books** have both pages prepared for sketching. These last are always ruled in squares, with red and blue ink, the sides of the squares being about one-quarter of an inch long. See Fig. 45, Pl. VII, in which full and broken lines have been substituted respectively for the blue and red rulings. The right-hand pages of compass and transit books are marked with a red line down the middle, and are ruled sometimes in squares, but quite as frequently in rectangles whose height is nearly twice their width. The latter form is unsuitable for sketching topography; the distortion arising from the difference of scale is more or less distracting to the topographer, and is apt to lead to unsatisfactory work.

193. The *Lead Pencil* for field sketching should be a moderately hard one, kept well sharpened. It should be used with firm strokes, producing slightly indented lines on the paper, whose traces will not be lost by accidental erasure. No line, be it straight or curved, should be drawn at random, but always as the result of a definite purpose.

194. In beginning to practise field sketching, the pupil must

* Ruled paper for topographical sketching is sometimes prepared in pads, or sketching blocks;—a very convenient arrangement whereby each sheet, on the completion of the sketch, can be detached and stored away in a waterproof bag, or satchel.

aim at accuracy, neatness, and celerity, and at the cultivation of a quick perception, and of a habit of decisive thinking in connection with all questions arising in his work. As his experience increases he will be able to discriminate promptly between features and measurements of different degrees of importance, and to indicate such distinctions in an off-hand way in the field-book ; but at the outset he had better limit himself, in this particular, to the discrimination between existing lines or boundaries and auxiliary lines, the former being represented by full lines, the latter broken or dotted.

The beginner should avail himself of every opportunity for testing and perfecting his ability to estimate horizontal and vertical distances and the inclination of slopes. He must also strive to acquire the faculty of noting quickly, and with precision, the magnitude, outlines and principal features of every object looked at, and to learn to project them on paper with the free hand, neatly and accurately, according to some assumed scale. He should accustom himself to direct, at will, his line of vision horizontally (this is greatly facilitated by an erect carriage of the head and body), and to follow with the eye contours on hill-sides, not only from his own level, but also when he is above or below them.

195. *Instruments*.—For the purpose of correcting his judgment, the pupil will need a *hand level*, or, still better, *Abney's combined level and clinometer*, whereby to trace level lines and measure vertical angles, and if, during his preliminary training, he can afford the expense, he should add to his outfit an *aneroid barometer* (3-inch face), for measuring altitudes, and a *prismatic compass*, or an ordinary pocket compass, wherewith to test his estimates of bearings or directions. A *pedometer*, for counting the topographer's steps or paces, will be found of great convenience. These instruments should be of the best quality.

Other suitable but more costly instruments not as generally known and used as the preceding, are : the *box sextant*, for measuring angles ; and for the measurement of the lengths of visual lines, Smyth's, Struve's, or Adie's *reflecting telemeter*, Gautier's *telemetrical telescope*, and Pratt's *range finder*. A description of these instruments and of the modes of using them belongs to treatises on surveying, and will not be attempted here.

196. Distances passed over by the topographer, if not previ-

ously determined, are usually ascertained by pacing, the number of steps being either counted or read from a pedometer. It is important, therefore, that he should know the average lengths of his step on a level, in ascending a slope, and in descending, and the variation of the paces from these averages at different rates of speed.

197. In order to ascertain distances from the observer to inaccessible or remote objects, it is often of great convenience to know the distance from the eye to a ruler held either vertically or horizontally at arm's length. The space on a rule, thus held, which covers the known height or length of an object in the field, is, then, the first term of a proportion of which, the height or length of the object covered, the distance of the rule from the eye, and the distance to the observed object, are, respectively, the second, third, and fourth terms. The latter distance can thus be calculated. Otherwise, a short scale may be constructed, the extreme length of which will cover exactly, let us say, 10 feet, at a distance of 100 feet from the observer. If this scale be divided into 10 equal parts, each part, therefore, representing 1 foot at a distance of 100 feet, the computation of distances by its aid can, in many cases, be performed mentally. Thus, if a tree or a house, estimated from its appearance to be 36 feet high, is covered by $1\frac{2}{3}$ spaces on the scale, its distance from the observer will be approximately $\frac{36 \times 100}{1\frac{2}{3}} = 3,000$ feet.

198. Field sketches must be drawn in plan. One of the greatest difficulties, therefore, which the beginner will probably encounter will arise from the distortion of objects by perspective, and from the apparent diminution of their dimensions. A knowledge of the rules of perspective and some proficiency in free-hand drawing will aid him materially in reducing the field of view to a plan drawing. Yet even with these aids he will frequently find himself at a loss for the means of estimating the unfamiliar dimensions of even the commonest objects, unless he shall have previously acquainted himself with some of the more constant proportions and dimensions of buildings and other structures; with the growth, sizes, and characteristic outlines and colors of different kinds of trees; and with the dimensions of many of the apparently unimportant details found in a landscape. The pupil may be surprised to learn that the length of a fence panel, or the sweep of a well, may assume great

importance in the eyes of a topographer, but such, nevertheless, is the fact.

199. *The inclination of a slope* is best estimated from a side view ; and an endeavor should, therefore, be made to obtain this view of a declivity when, for any reason, it becomes necessary to trust to the eye alone. Adjacent and distant hills often present some of their parts to view in profile. As a general rule advantage should be taken of all such favorable opportunities, and the observed inclination and direction of the slope should be noted, together with the name or other designation of the hill, its approximate distance and bearing from the point of view, and the vertical angle to its summit. If the hill is near the observer it may at once be located on the page in proper relation to other parts of the sketch ; but if it falls outside of the limits of the paper, the record, as indicated above, should be made on a separate page, with suitable reference to the point of view. Notes of outlying hills are often supplemented and rendered more intelligible by correct sketches of the field of view as seen in perspective.

200. It is particularly difficult to estimate the degree of a slope which either faces or recedes from the observer. Thus, a slope when viewed from its base usually appears less steep than when viewed from the top. A plain seen from an eminence appears to rise as it recedes from the observer ; while quite an appreciable descent from the foot of the eminence may seem to be perfectly level. The face of a hill when observed across an intervening valley seems to be much steeper than it really is, while the rear of the same hill, if visible, will almost surely be under-estimated in steepness. In all such cases a highly cultivated judgment is of the greatest value ; and this can be attained only by extensive practice, by repeatedly comparing estimates by the eye with actual measurements, by noticing in each case the kind of error, that is, whether of deficit or excess, and by carefully studying the probable cause of failure.

201. Hills are represented in field notes by means of either contours or hatchings. The former method presents decided advantages over the latter, in that it is more expeditious and, almost invariably, the more accurate method of the two. Moreover, a contour drawing, requiring only fine lines for the expression of slopes, is not as liable to become soiled or blurred as one which is shaded with hachures. Contours should there-

fore be used in preference to hatchings; but when it becomes necessary to distinguish two or more adjoining slopes inclined in different directions, short and light hatchings indicating the terminations of the slopes may be drawn over the contours.

202. No instructions can be given in regard to the methods of procedure in field sketching which will be suitable for all cases, and yet sufficiently specific to be of value. In almost every case the conditions are ever varying, calling for prompt decisions as to the best means of securing the information sought for. Measurement, estimation, platting, and sketching follow one another in rapid succession, or take any other order of precedence according as the opportunity for one or the other kind of work is offered. At times the topographer is compelled to depend entirely on very accurate measurement; as, for example, in running a line for a railroad through a settlement; at others, much of the work may be done from estimation. However, in order to give the student an insight into some of the methods used in practice two illustrations of different kinds of field notes have been provided in Plate VII.

203. *Fig. 44* is intended to illustrate the usual mode of field-sketching employed in surveys for lines of communication. The figure represents the right-hand page of a transit-book, ruled in squares, the ruling being represented by the dotted lines which have been substituted for the blue lines in the original to facilitate the engraving. For the same reason the centre line, which in this kind of field book always represents the instrumental line, has been drawn in black instead of red—the usual color. The instrumental line is staked out very accurately by the “transit party” into intervals of 100 feet, and is then gone over by the leveling party, and thus, the alignment, deflections, and horizontal distances of the instrumental line, as well as the undulations of the ground along this line, are accurately determined. The proper place for the topographer is either with the leveling party or immediately in the rear of it. He will then be able to avail himself of the altitudes on the main line as given by the level, and will thus have the means, at once, of marking on the centre line of his note book the true positions of contours, which it will then be his duty to extend to the right and left by means of his own.

204. The record in transit or compass books should always

begin at the bottom of the page, progressing upwards, to provide for the sketching of objects on the right and left hand of the centre line of the survey in the same relative positions in the drawing, without having to invert the book. The figure, accordingly, represents near the bottom of the page the initial point of a survey beginning at the intersection of two streets of a settlement. The direction of the centre line is determined by the angles it makes with the axes of the streets, as well as by the magnetic bearing. These angles and the adjacent block lines are recorded by the topographer on his sketch. As the transit party advances, leaving numbered stakes in the ground, he records either from estimation or measurement the inclinations and directions of slopes on either side of the stations, and locates prominent objects near the line by intersecting bearings, pacing, or any other means which may be most convenient and appropriate to the importance of each particular object ; * and sketches, merely from estimates by the eye, distant or unimportant features. Meanwhile, the level notes which will have been taken will enable him to locate the contours on the main line. This being done, he will be able to prolong them by the aid of the distances and altitudes already recorded, or by interpolating them on the lines of inclination previously noted.

305. The interpolation of contours can be accomplished very rapidly by keeping in mind the approximate rule that, if a slope of 1° be represented by a rise of 1 foot in 60 feet of horizontal base—which is near enough for the purpose—the bases corresponding to a rise of 1 foot for other inclinations up to 20° can be found, nearly enough for the topographer's purposes, by dividing 60 by the degree of the inclination. According to this rule a slope of 15° , for example, will rise at the rate of 1 foot in 4 feet (see table, *Par.* 117). Knowing, then, the vertical intervals between the contours (usually 5 or 10 feet), the angle of the

* When he is provided with sheets of topographer's paper, a most convenient arrangement in comparatively open ground is to fasten a sheet to a small board which is supported on the head of a very light tripod by means of an ordinary ball and socket joint and spindle, thus forming a miniature plane table which, in surveys of this kind, gives more accurate results than the pocket-compass, and admits of fully as expeditious work. The tablet, being set up approximately over convenient stations on the main line, enables the topographer to locate on paper prominent points on either side of the instrumental line, from which he can extend his work by estimates and sketches. The vertical angle should be taken to every point located by means of the tablet.

slope, and the value of each division on the paper, the interpolation of contours becomes a very simple operation. The ratios corresponding to other inclinations are for 25° , $\frac{1}{4}$; for 30° , $\frac{3}{4}$; for 40° , $\frac{1}{2}$; and for 45° , $\frac{1}{3}$.

206. In measuring the slopes of hill-sides, the line of sight need not be arrested at every slight change of inclination. The general trend is to be taken with the instrument, and slight irregularities are to be indicated afterwards by appropriate variations in the spaces between the contours. See contours in Fig. 44 from the intersection of streets toward the pond; also contours near the middle of the figure on the line marked (5°). The slope on this line should have been marked (4°), instead of (5°), the latter being a mistake in inking the plate.

207. A plug, or transit point, is indicated at station 14, where the direction of the line is changed. The usual manner of determining the change of direction is by measuring the angle which the new line makes with the preceding one produced. This angle is termed the *angle of deflection*, and is inscribed on the topographer's page, as indicated in the figure. The letter R (right) or L (left) should in all cases be appended to the inscription, to indicate the direction in which the angle has been turned off, thus avoiding all ambiguity. Some note-men use the short line for indicating, by its own direction with regard to the centre line, that of the deflection; while others regard it as standing in place of the prolongation of the preceding instrumental line. The first class would interpret the figure as indicating a deflection to the right, the second to the left, unless appropriate letters were added to the inscription.

At such points, the field notes of consecutive instrumental lines should be separated by a heavy line drawn across the entire page. Sometimes an interval of a line or two is left between this line and the succeeding sketching. The sketching of the new part is then executed in proper relation to its own centre line and independent of the preceding drawing, excepting that the order of the altitudes of contours should be preserved. The sketches of consecutive sections may be extended into the unoccupied intervals or they may be made to lap over each other slightly at the edges, where, in platting the map, the deflection of the main lines will naturally cause a break or vacant place in the topography.

208. The other illustration (Fig. 45) is intended to illustrate the manner of determining and drawing contours in the field in topographical surveys of hilly regions. The paper usually employed is that described in the foot-note of *Par.* 192.

The upper part of Fig. 45 represents the head of a valley densely wooded, which has been surveyed on the exterior by transit lines (shown by the lines terminating at the points where the altitudes have been recorded), from which lines the topographer has run courses, with the pocket compass and level or clinometer, into the interior woodland, in order to determine the undulations of the ground. These courses are indicated in the figure by fine right lines. His method of procedure is somewhat as follows:—Having previously drawn the transit lines and marked the elevations at their extremities, and having interpolated the contours, extending them within the limits of correct estimation, he then selects some convenient transit station,—for example, the one near the left of the figure, marked 431.6,—and runs as long a line as he can along a measurably uniform part of the slope, and in the direction of the area to be explored, taking the bearing, length, and vertical angle. From these data, the line is platted on the topographical sheet in proper relations to the transit lines, and the difference of level between the extremities calculated. This difference is sometimes found directly by means of a hand level.

Knowing, then, the elevations of these extreme points, the contours are readily interpolated on the paper, and are then extended as far as possible laterally, the directions of occasional contours being guided, or checked, when they are of considerable length, by sights with the level and compass. The contours should be numbered at convenient intervals for the guidance of the topographer as well as of the draftsman who is to construct the map. From the extremity of the line just located, another line is run, similarly, in any direction that promises best to develop the characteristic inequalities of the surface. It is drawn on the sheet, the elevation of its outer extremity is established and marked on the paper, and contours are interpolated and extended as before. Thus, course after course is run, seeking always to secure the longest lines and to command the best view of the places where changes of inclination occur.

Each set of lines should, whenever it is admissible, be run to

a closure with some station previously determined on the paper—a transit station if possible—for the purpose of verification. The topographer has thus the means of applying to his work all necessary verifications and corrections in the field, before sending his notes to the draughting-room.

209. The lower part of Fig. 45 is designed to illustrate the mode of employing auxiliary lines of the same character as those described in the preceding case in connection with a system of primary lines forming squares. The corners of the squares are indicated in the figure by a distinctive mark. The primary lines are accurately staked out, usually with the transit and chain, and are run over with the level, or the elevations are determined in part or in whole by means of vertical angles. The intersections of these lines and prominent points in their profiles form points of departure for the topographer's lines which are run in different directions for the purpose of tracing the contours within the squares.

210. Contours should be drawn on the topographer's sketches in very clear and firm (but not stiff) lines. They should be free from all vagueness as regards direction or emphasis, and should express as definitely as possible the topographer's idea of the conformation of the ground. The work of transferring the contours to the map will be rendered more satisfactory, and the value of the map will be increased, by an observance of these instructions.

APPENDIX A.

PROBLEMS CONNECTED WITH THE REDUCTION, ENLARGING, AND COPYING OF MAPS OR PLANS.

PROBLEM 1. *To Construct a Square that shall be a Multiple of any given Square.* Let $A B C D$ (Fig. 54) be the given square, and let it be required to construct a square that shall contain 2, 3, 4, etc., times its surface. Draw the diagonal $B D$, and make $B a$ equal to $B D$ —then the square described upon $B a$ will be double the square $A B C D$. Lay off $A E$, equal to $B a$, and draw $B E$, then the square described upon $B E$, or $B d$, will be three times the square $A B C D$. In the same manner lay off $A F$ equal to $B d$, and the square described upon $B F$, or $B c$, will be four times the square $A B C D$, and so for any multiple of the square $A B C D$.

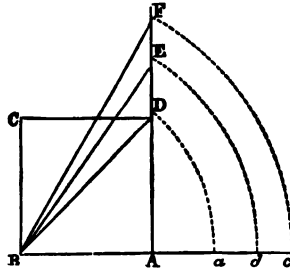


FIG. 54.

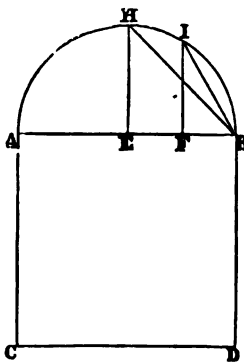


FIG. 55.

PROBLEM 2. *To Construct a Square that shall be equal to $\frac{1}{2}$, $\frac{1}{4}$, etc., of any given Square.* Let $A B C D$ (Fig. 55) be the given square. On $A B$, as a diameter, describe the semicircle $A H B$, and erect, at the centre E , the perpendicular $E H$. Draw $B H$, and it will be the side of a square equal to one-half of $A B C D$. Lay off $F B$, equal to one-fourth of $A B$, and erect the perpendicular $F I$, then the square described upon $I B$ will be equal to one-fourth of $A B C D$. In the same manner, a square may be constructed, equal to any part of $A B C D$.

PROBLEM 3. *To Construct a Square that shall be in any proportion to a given Square.* Let $A B C D$ (Fig. 56) be the given square. It is required to construct a square which shall be to $A B C D$ as 2 is to 5. Upon the side $A B$ as a diameter, describe the semicircle $A G B$, and divide the line $A B$ into five equal parts. At the second point of division, erect the perpendicular $F G$, and join $A G$. The square described upon $A G$ will be to the given square $A B C D$ as 2 is to 5.

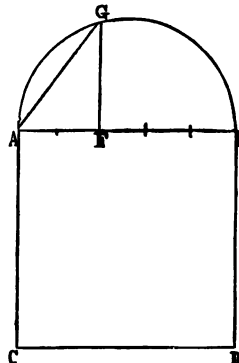


FIG. 56.

PROBLEM 4. *To Construct upon a given Base, a Rectangle, which shall be similar to a given Rectangle.*

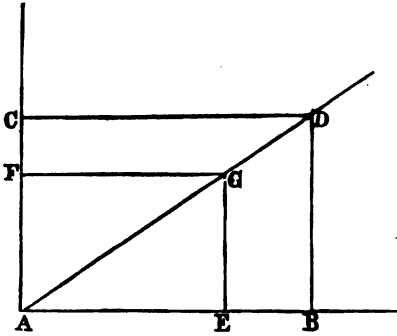


FIG. 57.

Let $A E F G$ be the given rectangle. It is required to construct upon the base $A B$, one that shall be similar to $A E F B$. Prolong $A E$, and lay off the given base from A to B . Draw the diagonal $A G$, and prolong it indefinitely. Erect a perpendicular to $A B$, at B , and at the point D , where it intersects the prolonged diagonal, let fall $D C$, perpendicular to $A F$ produced. Then $A B C D$ will be similar to $A E F G$. All rectangles having their diagonals in the same line are similar.

angles having their diagonals in the same line are similar.

APPENDIX B.

ON THE METHOD OF DRAWING BY THE USE OF REFERENCES AND PROJECTIONS ON A SINGLE PLANE OF PROJECTION.

To *project a point* upon any plane is to let fall from the point a perpendicular to that plane; the foot of the perpendicular is the *projection* of the point.

Projections.—The projection of a point upon a horizontal plane is that point of the plane directly beneath the given point. The lines which project the points of a straight line constitute a plane, and this plane is called the *projecting plane* of the line. Its intersection with the plane on which the projection is made is a straight line, and is the projection of the given line. Hence, if the projections of two points of a straight line are known, the projection of the line can be drawn. The projection of a curved line will be a curve excepting when the given curve lies in a vertical plane, when its projection will be a straight line.

The plane on which the projection is made is called the *Plane of Projection*, and the lines which project points are called *projecting lines*. In what follows, the plane of projection will be taken *horizontal*, and all objects represented are supposed to lie *above* that plane.

Plane of Reference.—A horizontal plane of projection, relatively to which the heights of a series of points or lines are considered, is called the *plane of reference* of those points or lines.

Reference.—The numbers expressing the heights of points above the plane of reference are called the *references* of the points, and are written on the drawing near to the projections of the points.

Line of slope.—The line of greatest descent of any plane or surface is called the *line of slope* of that surface.

Scale of slope.—Any equal divisions, marked upon a line of slope, indicating the *amount of descent* corresponding to any horizontal measurements, constitute a *scale of slope*.

Horizontal scale.—A scale of equal parts indicating the relative distances and positions of points situated in the plane of reference is called the *horizontal scale*. For each drawing there is but one horizontal scale; but there may be as many different scales of slope as there are various inclinations of lines or surfaces represented, and there must evidently be an established relation between every scale of slope and the horizontal scale.

From the above definitions it will be seen that a *point* is determined in

space when we know *its projection* on the plane of reference and its height above that plane, that is, *its reference*. Also, that a *straight line* is determined when we know (1st) *its projection*, *its scale of slope*, and the *reference of any point* of the line, or (2d) *its projection* and the *references of any two of its points*. A *plane* is determined when we know the *references and projections* of at least *three* of its points; or when its *line of slope* is known.

A few problems, illustrating the application of the simple principles above defined, will explain the use of this method of representing points, lines and surfaces by means of one projection only. It is chiefly useful where the heights to be represented are quite small in comparison with the horizontal extent of the surface; as in topographical drawing, "laying out" of terraces, earthworks, etc., and in "staking out" for constructions.

Any convenient scale of equal parts may be assumed as the horizontal scale of the drawing. For the following problems a scale of $\frac{1}{317}$ will be used.

PROBLEM 1. *To determine the scale of slope of a given line.* The projection of the line and the references of two of its points being given, the scale of slope depends on the relation existing between the projection of that part of the line included between these two points and the difference of their references. Let A E (*Fig. 58*) be the given line; A P and E R the projecting lines upon the plane of reference, P R, of the given points A and E. Let the reference A P be 9.4 yards, and E R, 5.8 yards, and let the projection P R be 16.2 yards. Lay off A B = 1 yard, and draw B C and D E parallel to P R. For the ratio between the difference of reference and the projection we have

$$\frac{A B}{C B} = \frac{A D}{E D} = \frac{A P - E R}{P R} = \frac{9.4 - 5.8}{16.2} = \frac{3.6}{16.2} = \frac{2}{9} = 0.2222.$$

For the projection of the line A C, the difference of the references of the ends being unity, we have

$$P Q = B C = \frac{P R \times A B}{A P - E R} = \frac{16.2}{3.6} = 4.5 \text{ yards.}$$

For the projection of any line, C E, the difference of reference, B D, being known, we have

$$Q R = \frac{P R \times B D}{A P - E R} = \frac{16.2 \times 2.6}{3.6} = 11.7 \text{ yards.}$$

For the difference of reference of a line whose projection is unity, we have

$$\frac{A B}{B C} = \frac{A P - E R}{P R} = \frac{3.6}{16.2} = 0.222 \text{ yards.}$$

For the difference of reference of a line C E, the projection Q R being known, we have

$$B D = \frac{(A P - E R) Q R}{P R} = \frac{3.6 \times 11.7}{16.2} = 2.6 \text{ yards.}$$

The length in space of any portion, A E, of a line may be found from its references and projection by means of the formula

$$A E = \sqrt{A D^2 + D E^2} = \sqrt{(16.2)^2 + (3.6)^2} = 16.6 \text{ yards.}$$

The unit of the scale of slope is the projection of the line whose difference of reference is unity, and in this case has been found to be 4.5 yards. To make the scale of slope, first find the projection of a point M whose reference is an integer; in this case MS = 9, and we have the projection

$$SR = \frac{PR(MS - ER)}{AP - ER} = \frac{16.2(9 - 5.8)}{9.4 - 5.8} = 14.4 \text{ yards.}$$

Lay off by scale RS = 14.4 yards, call S 9, and lay off 9 - 8, 8 - 7, etc., each equal to 4.5 yards. The scale of slope is complete.

PROBLEM 2. *To determine the slope of a line, having given its projection and the references of two of its points.* The slope of a line is the angle which it makes with its projection on the plane of reference. It is expressed by dividing the difference of reference of any two of its points by the number expressing the length of the projection between these two points. Thus in Fig. 58 the slope is

$$\frac{AB}{BC} = \frac{AD}{DE} = \frac{AP - ER}{PR} = \frac{3.6}{16.2} = \frac{2}{9};$$

and this would be called "a slope of 2 upon 9," that is, 2 vertical to 9 horizontal.

This expression for the slope is also the *tangent of the angle*; and if it is desired to know the number of degrees, minutes and seconds in the angle, the fraction must be reduced to a decimal of five or six places; and in a table of natural tangents, the degrees, etc., will be found opposite to that decimal (or mixed number, as the case may be). In the case given, the tangent of the angle is $\frac{2}{9} = 0.22222$, and the corresponding angle is $12^\circ 31' 44''$.

In general, if r represents any difference of reference, l the corresponding length of projection, s the slope, and L the length of the line in space, we have

$$s = \frac{r}{l}; \quad r = sl; \quad l = \frac{r}{s};$$

$$L = \sqrt{r^2 + l^2} = l \sqrt{s^2 + 1} = \frac{r}{s} \sqrt{s^2 + 1}.$$

It is evident that the value of a unit of the scale of slope is l when $r = 1$; that is, it is $\frac{1}{s}$. A line is thus known when its projection, its slope (or its scale of slope), and the reference of one point are given.

The reciprocal of the slope, that is, the projection divided by the difference of reference, is usually known as the *slope ratio*.

PROBLEM 3. *Knowing the projection of a point on a given right line, to find the reference of that point.* Let the line ND (Fig. 62) be given by its projection and by the references AP and BR of two of its points, A and B. It is required to determine the reference MQ of the point M, whose projection, Q, is given. Let AP = 8; BR = 4.7; PR = 10.5; and, by measurement, PQ = 7. Then by Prob. 2 we have

$$s = \frac{r}{l} = \frac{AP - BR}{PR} = \frac{8 - 4.7}{10.5} = \frac{1.1}{3.5}$$

Whence, when $r = AC$ and $l = CM = PQ = 7$,

$$AC = r = sl = \frac{1.1}{3.5} \times 7 = 2.2 \text{ yards,}$$

and $MQ = AP - AC = 8 - 2.2 = 5.8$ yards.

When the given point N is higher than A , we have, when the projection $OP = l = 3.1$,

$$AF = r = sl = \frac{1.1}{3.5} \times 3.1 = 0.97,$$

and $NO = AP + AF = 8 + 0.97 = 8.97$ yards.

When the scale of slope is given it is only necessary to find, from the subdivisions of the scale of slope, the distance of the point from either one of the principal divisions between which it lies; as in *Fig. 63*, which represents a scale of slope. The point D , marked zero, is evidently the point where the given line pierces the plane of reference.

PROBLEM 4.—*Knowing the reference of a point on a given right line, to find the projection of that point.* Knowing the reference of the point M (*Fig. 62*), $MQ = 5.8$, and the line AB by references and projection, it is required to determine the projection Q of that point; in other words, to calculate the length of PQ . We have $AC = AP - MQ = r = 2.2$; and from Prob. 3, $s = \frac{1.1}{3.5}$; hence

$$PQ = l = \frac{r}{s} = 2.2 \times \frac{3.5}{1.1} = 7 \text{ yards.}$$

If the scale of slope were known, the projection corresponding to the given reference could be taken from the scale, as in the last problem.

In order to obtain the point D in which the line pierces the plane of reference we have, since the reference of D is zero,

$$r = AP = 8; s = \frac{1.1}{3.5};$$

$$\text{whence, } PD = l = \frac{r}{s} = 8 \times \frac{3.5}{1.1} = 25.45 \text{ yards.}$$

PROBLEM 5. *Through a given point to draw a line parallel to a given line.* It is evident, from the definitions already given, that two lines will be parallel when their projections are parallel and their slopes (or scales of slope) are equal. *Fig. 64* shows the projection and scale of slope of a line drawn parallel to AB through a point whose projection is C and reference 7.5. The parallel dotted lines connecting points having equal references are evidently horizontal lines lying in the plane containing both the given and required lines.

The representation of a Plane Surface.

Any plane not parallel to the plane of reference will intersect it in a straight line called *the trace* of the plane; and the angle made by the oblique plane

with the plane of reference will be measured by (that is, it will always be the same as) the angle formed by two lines, one in each plane, meeting the intersection of the planes at the same point of that intersection and perpendicular to it. A line drawn in an oblique plane, perpendicular to the trace of the plane, will form, with the projection of the line, this measuring angle; and the line so drawn in the oblique plane will be *its line of greatest descent*, or *its line of slope*.

A plane is determined if its line of slope is known. To distinguish the scale of slope of a plane from that of a line, the former is represented by a double line (*Fig. 65*). If a plane is *horizontal*, it is represented by the reference of any one of its points, as the references are the same for all of its points. If a plane is *vertical*, it is represented only by its trace on the plane of reference. The direction of the trace of a plane is known when that of its line of slope is known, and, conversely, the latter is known if the former is given, since those two lines are always perpendicular to each other.

PROBLEM 6. *To pass a plane through three given points, and determine its scale of slope.*—Let A, B and C (*Fig. 66*) be the three points, whose given references are respectively 12, 8.5 and 6. Join any two of the points, as A and C, and construct the scale of slope of the line A C. Find upon it the point H whose reference is 8.5, and join it with B. This line H B will evidently be a horizontal of the required plane. Then F G, perpendicular to B H, will be a line of greatest descent of the plane, and upon it the scale of slope can be constructed. The point 8.5 of the scale is known, and by drawing through A the horizontal A K, parallel to B H, the point 12 of the scale will be determined, and the scale may then be constructed.

PROBLEM 7. *Through a given point and a given line, to pass a plane and determine its scale of slope.* Find upon the given line the point which has the same reference as that of the given point. Join it with the given point. This line will be a horizontal of the required plane. Parallel to that draw a line through any other point of the scale of the given line. This line will be another horizontal of the required plane. The scale of slope of the plane can then be constructed as in Prob. 6.

PROBLEM 8. *To pass a plane through a given point, parallel to a given plane, and determine its scale of slope.* Let B C (*Fig. 67*) be the scale of slope of the given plane, and A (5.7) be the given point. Since the planes are to be parallel, the lines of greatest descent will be parallel. Therefore the scales of slope will be parallel. Since they also make the same angle with the plane of reference, their scales of slope will be equal. Drawing through A a line parallel to B C, it will be the required line of slope, upon which, starting with the given reference 5.7, the scale may be constructed.

PROBLEM 9. *To determine the projection and scale of slope of the line of intersection of two given planes.* Let A B and C D (*Fig. 68*) be the scales of slope of the given planes. Take upon A B the points G and K at any convenient distance, and upon C D the points H and L having the same references respectively as G and K. Draw G O and K P perpendicular to A B, and H O and L P perpendicular to C D. These lines will be horizontals of the given planes, and their intersections O and P will evidently be points

common to the two given planes. Joining O and P will give the required intersection. The scale of slope will in this case be found by dividing O P into five equal parts. The zero point of the line of intersection coincides with the intersection of the perpendiculars at A and C.

PROBLEM 10. *To determine the projection and the reference of the point in which a given line intersects a given plane.* If any plane be passed through the given line, and the intersection of that plane with the given plane be found, it is evident that the given line will pierce the given plane at the point where it meets that intersection. A convenient plane to use for this problem is the plane which has the same scale of slope as the given line. Let A B (*Fig. 69*) be the scale of slope of the given plane, and F G that of the given line. Make F G the scale of slope of the auxiliary plane, and find its intersection C D with the given plane, by Prob. 9. The point O, in which the given line F G meets that intersection, is the point in which F G pierces the plane A B.

APPENDIX C.

TABLE GIVING THE LENGTHS OF ONE DEGREE OF LATITUDE AND LONGITUDE IN METRES AND STATUTE MILES AT DIFFERENT LATITUDES. ALSO THE LENGTHS OF THE RADII OF DEVELOPED PARALLELS OF LATITUDE, AND THE ANGLES AT THE APICES OF THE TANGENT CONES FOR 10° OF LATITUDE (prepared from the Coast Survey tables).*

LAT.	1° OF LATITUDE.		1° OF LONGITUDE.		RADIUS OF PARALLEL IN METRES AND STAT. MILES.	FOR 10° OF LONGITUDE.
	IN METRES.	IN STAT. MILES.	IN METRES.	IN STAT. MILES.		
°					14,381,780†	° ' "
24	110,747	68.812	101,740	63.216	8,905.	4 04 02.5
					13,684,530	
25	110,762	68.822	100,938	62.717	8,502.8	4 13 34.3
					13,083,990	
26	110,776	68.831	100,106	62.200	8,129.7	4 23 01.4
					12,524,960	
27	110,792	68.840	99,243	61.664	7,782.3	4 32 23.7
					12,002,960	
28	110,808	68.850	98,350	61.109	7,458.0	4 41 41.0
					11,524,770	
29	110,824	68.860	97,427	60.536	7,160.9	4 50 53.1
					11,055,200	
30	110,841	68.871	96,475	59.944	6,869.1	5 00 00.0
					10,623,179	
31	110,858	68.881	95,493	59.334	6,600.7	5 09 01.4
					10,215,570	
32	110,875	68.892	94,482	58.706	6,347.4	5 17 57.1
					9,830,067	
33	110,892	68.903	93,442	58.060	6,107.9	5 26 47.0
					9,464,760	
34	110,911	68.914	92,374	57.399	5,880.9	5 35 31.0
					9,117,882	
35	110,929	68.925	91,277	56.715	5,665.4	5 44 08.8

* C. S. Report for 1859, Appendix No. 33, by J. E. Hilgard, Assistant (now Superintendent) Coast Survey.

† The upper figures give the radius in metres, the lower in statute miles.

LAT.	1° OF LATITUDE.		1° OF LONGITUDE.		RADIUS OF PARALLEL IN METRES AND STAT. MILES.	° FOR 10° OF LONGITUDE
	IN METRES.	IN STAT. MILES.	IN METRES.	IN STAT. MILES.		
°					8,787,972	° ' "
36	110,947	68.936	90,153	56.016	5,460.4	5 52 40.3
					8,473,840	
37	110,966	68.948	89,001	55.300	5,264.9	6 01 05.3
					8,173,042	
38	110,985	68.960	87,822	54.568	5,078.3	6 09 23.8
					7,885,875	
39	111,003	68.971	86,616	53.819	4,899.9	6 17 35.5
					7,610,788	
40	111,023	68.983	85,384	53.053	4,728.9	6 25 40.4
					7,346,915	
41	111,042	68.995	84,125	52.271	4,575.5	6 33 38.1
					7,093,423	
42	111,061	69.007	82,841	51.473	4,407.5	6 41 28.7
					6,849,560	
43	111,080	69.019	81,531	50.659	4,255.9	6 49 11.9
					6,614,648	
44	111,100	69.031	80,197	49.830	4,110.0	6 56 47.7
					6,388,064	
45	111,119	69.043	78,837	48.985	3,969.2	7 04 15.6
					6,169,244	
46	111,139	69.056	77,454	48.126	3,833.2	7 11 36.2
					5,957,663	
47	111,158	69.068	76,047	47.251	3,701.8	7 18 48.7
					5,752,845	
48	111,178	69.080	74,616	46.362	3,574.5	7 25 53.2
					5,554,355	
49	111,197	69.092	73,163	45.460	3,451.2	7 32 49.6
					5,361,781	
50	111,216	69.104	71,687	44.543	3,331.5	7 39 37.6
					5,174,752	
51	111,236	69.116	70,189	43.612	3,215.3	7 46 17.3
					4,992,925	
52	111,255	69.128	68,670	42.668	3,102.3	7 52 48.4
					4,815,973	
53	111,273	69.139	67,129	41.710	2,992.4	7 59 10.9
					4,643,603	
54	111,292	69.151	65,568	40.740	2,885.3	8 05 24.6

The table is based on the following constants of Bessel's spheroid $\frac{1}{299.15}$ used in the Coast Survey.

Equatorial radius of the earth... $a = 6,377,897$ metres... $\log. = 6.80464346$.

Polar radii of the " ... $b = 6,356,079$ " ... $\log. = 6.80318928$.

Square of the eccentricity $\frac{a^2 - b^2}{a^2}$... $e^2 = 0.00667437$ metres; ... $\log. =$

7.8244104.

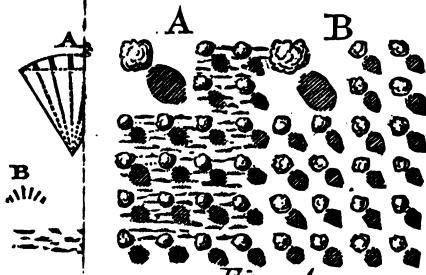


Fig. 4.



Fig. 5.

9.



Fig. 10.

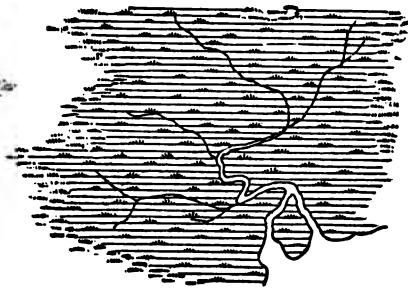
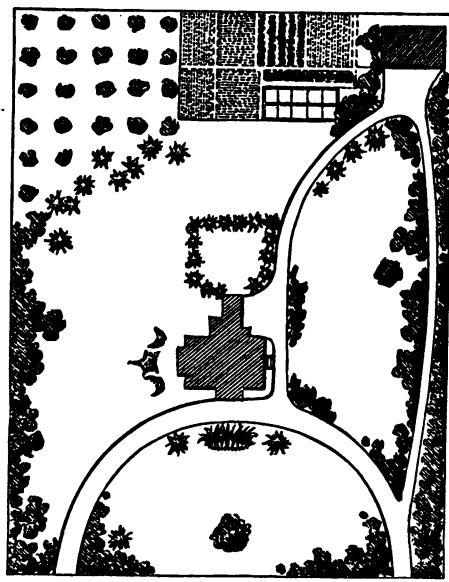


Fig. 14.



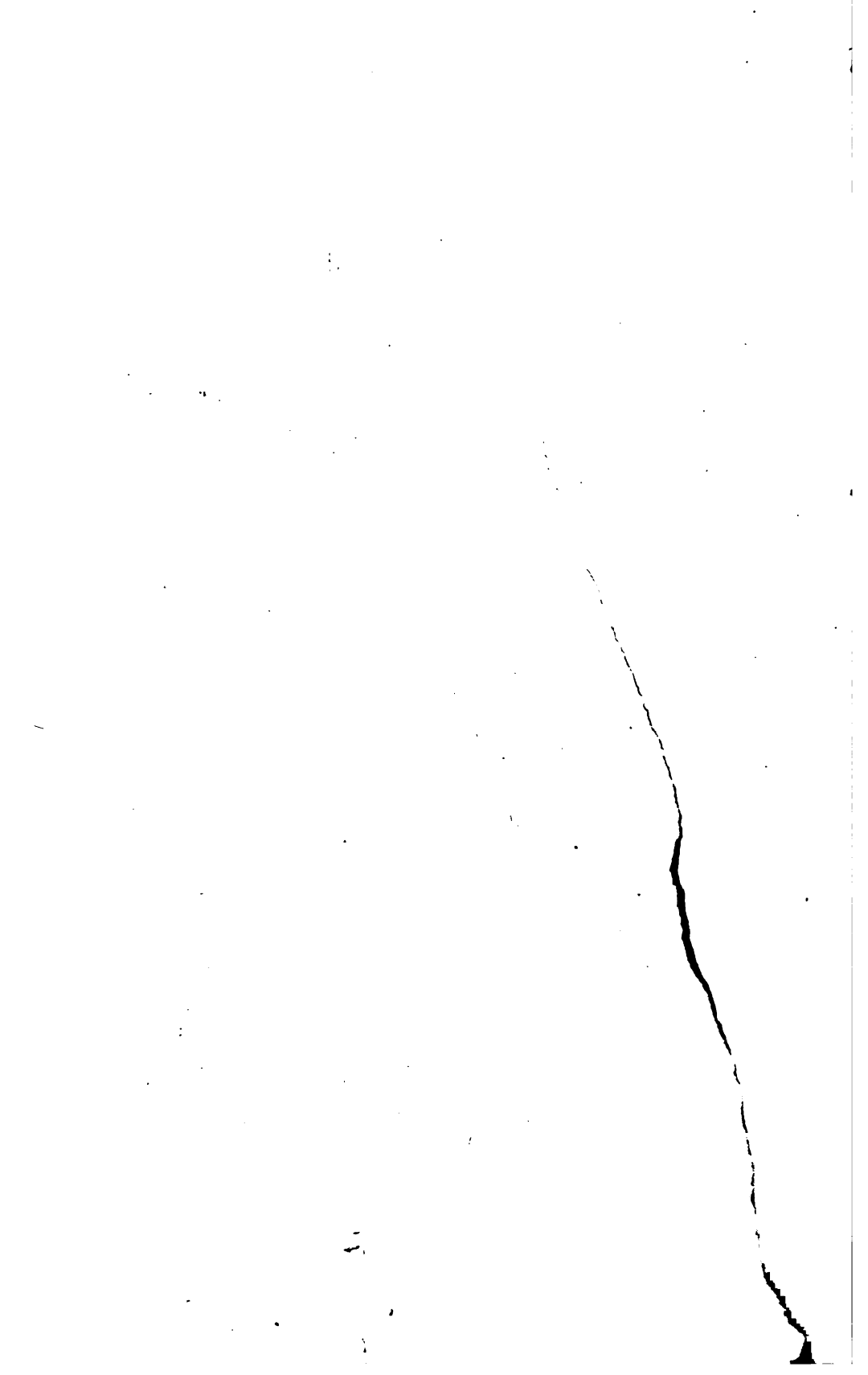


Fig. 18.

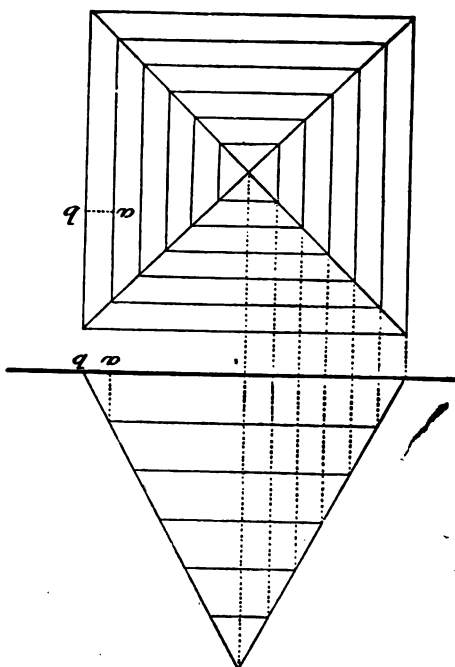
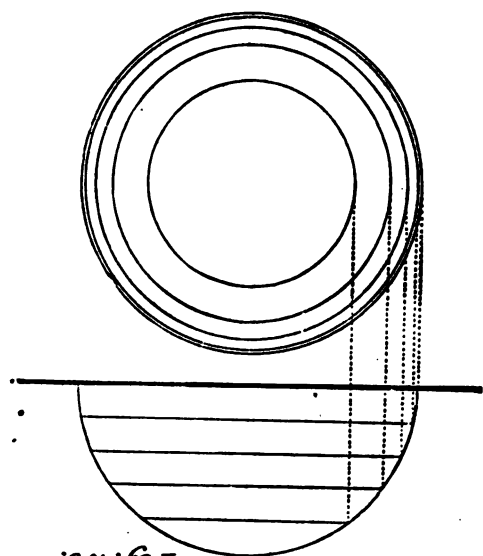


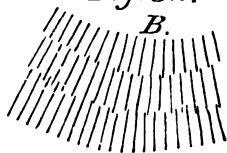
Fig. 20.



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Fig. 32.

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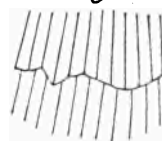


Fig. 35.

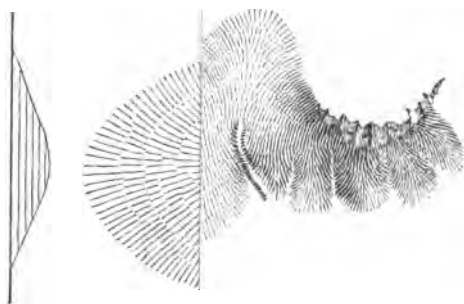


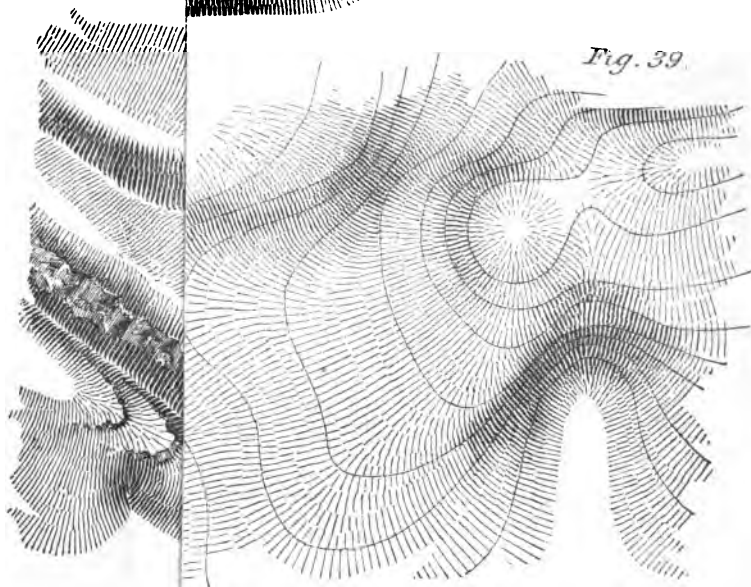
Fig. 36.



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Fig. 39.



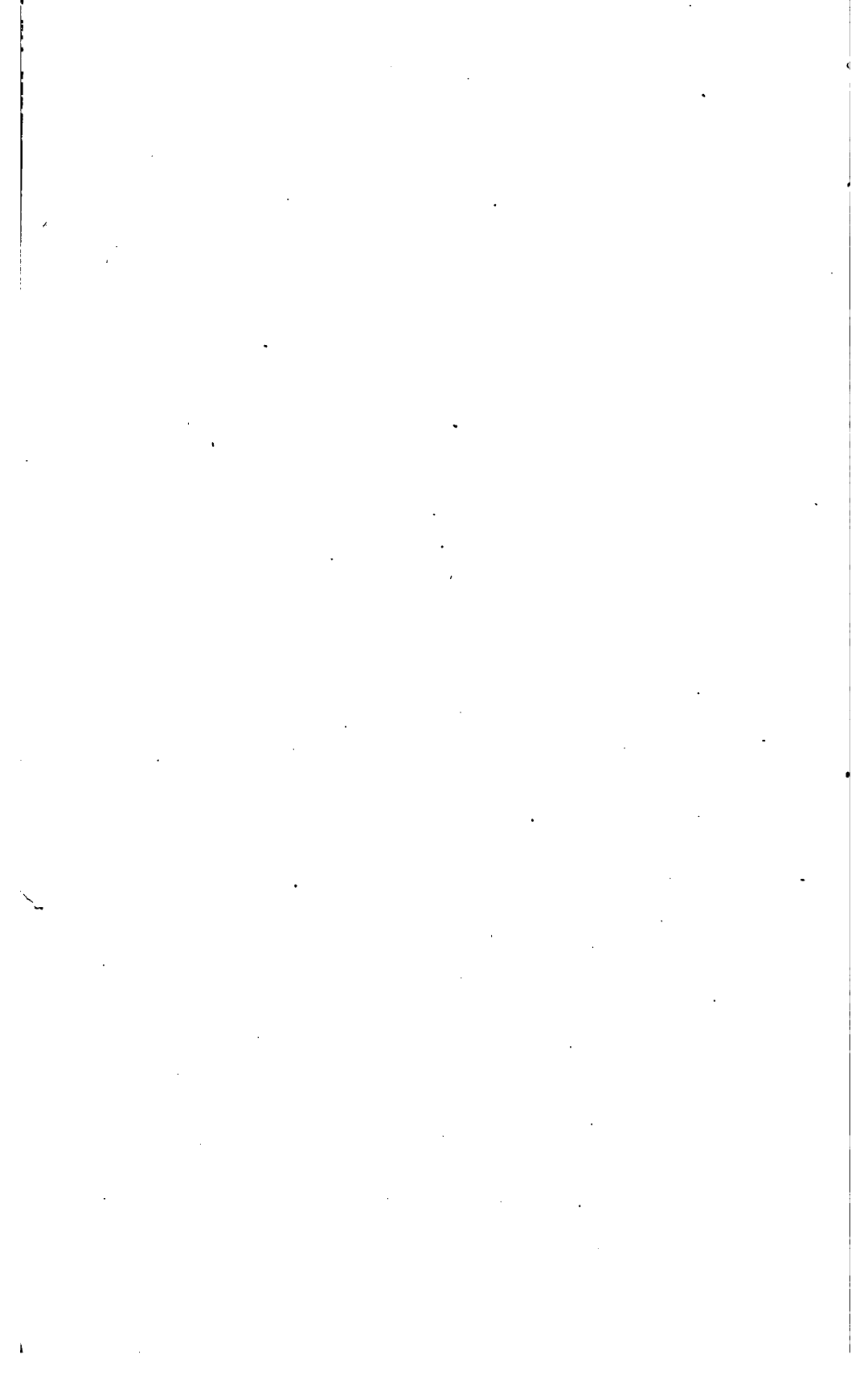
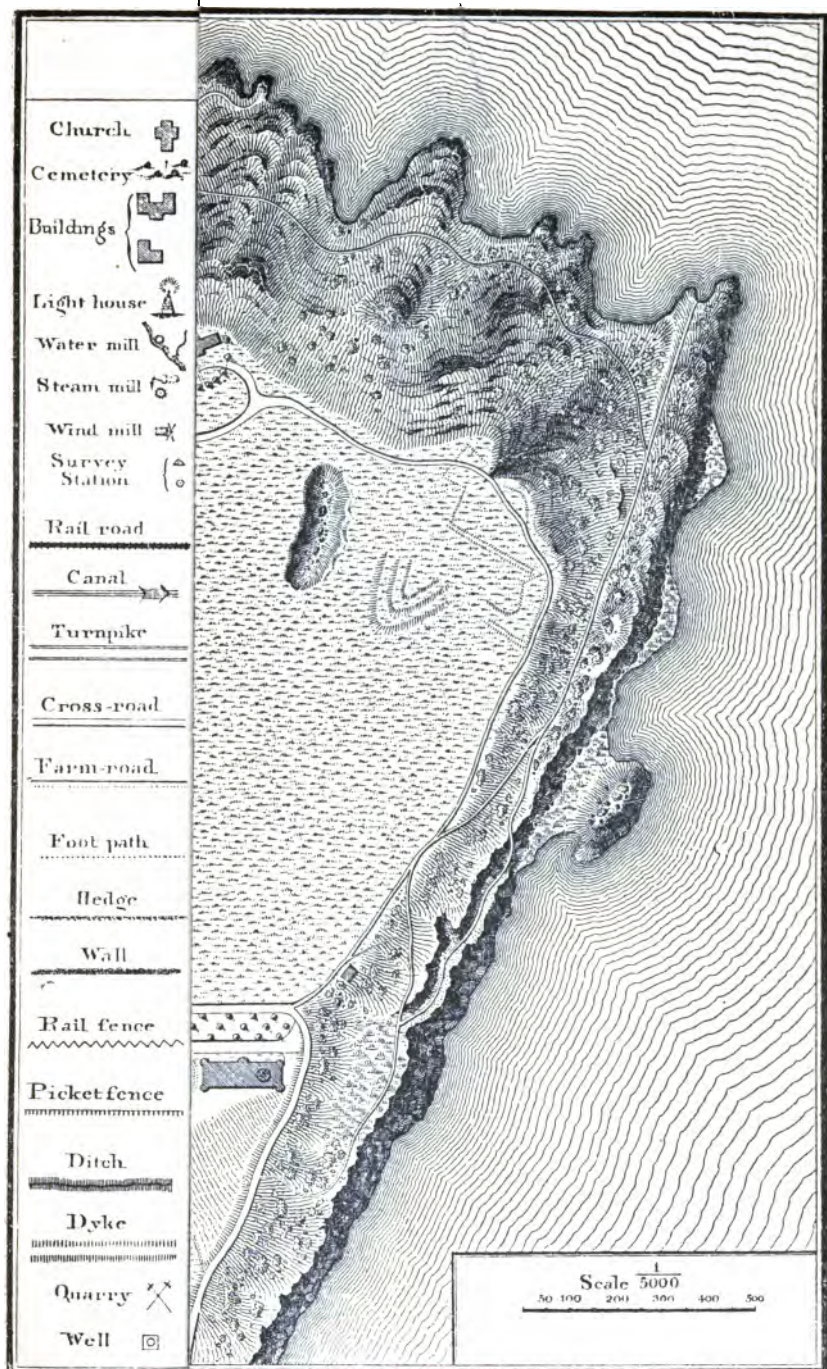
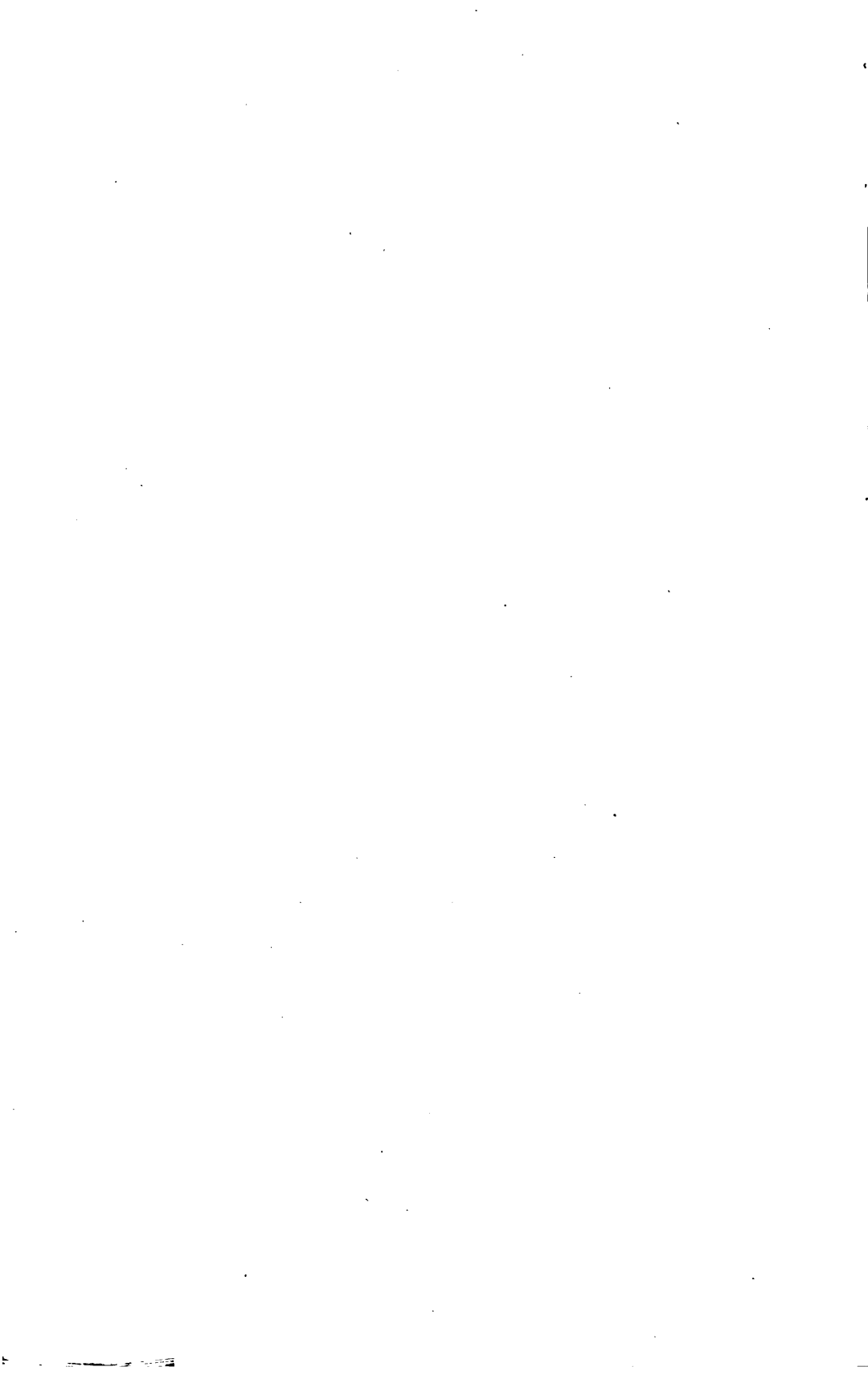
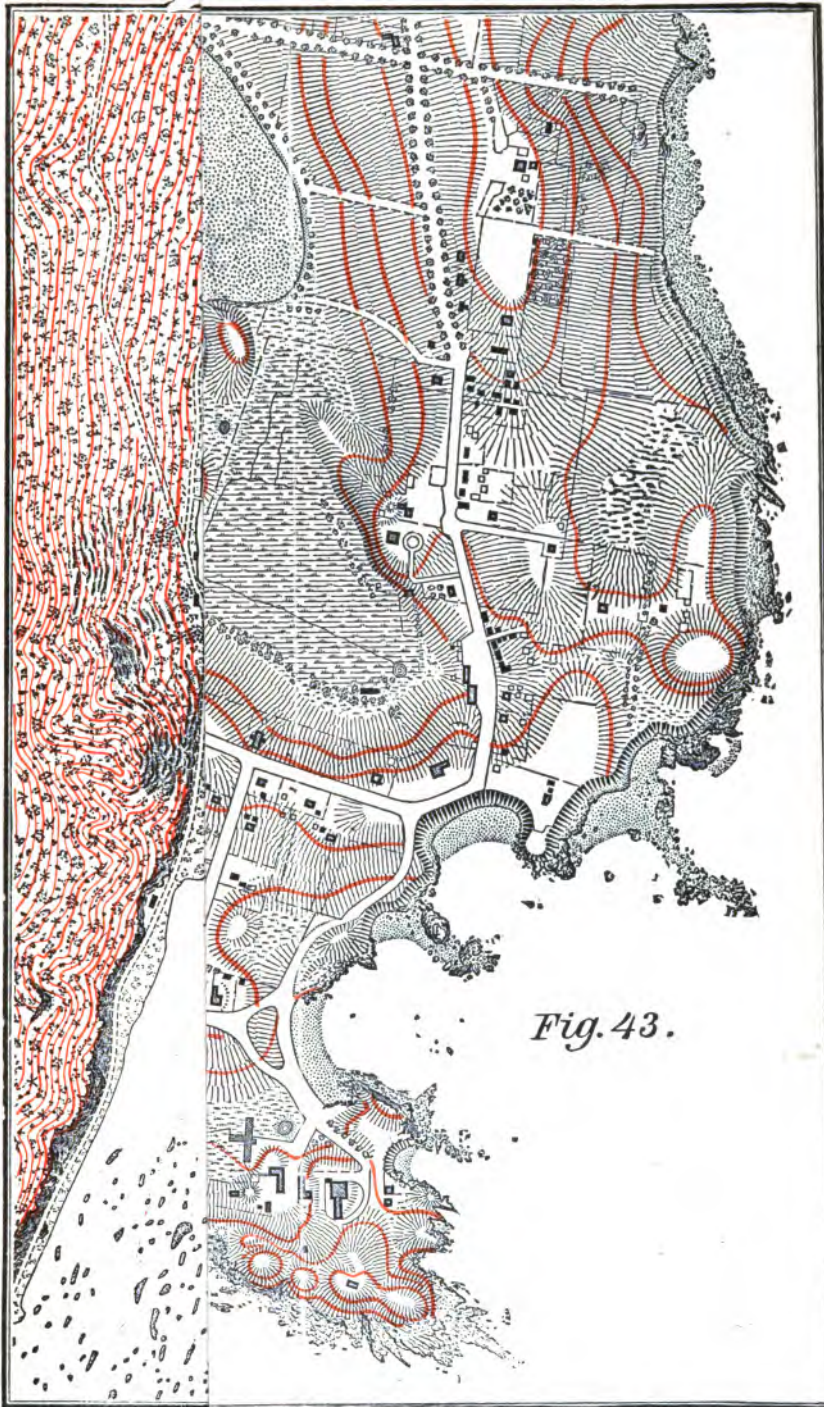


PLATE V.







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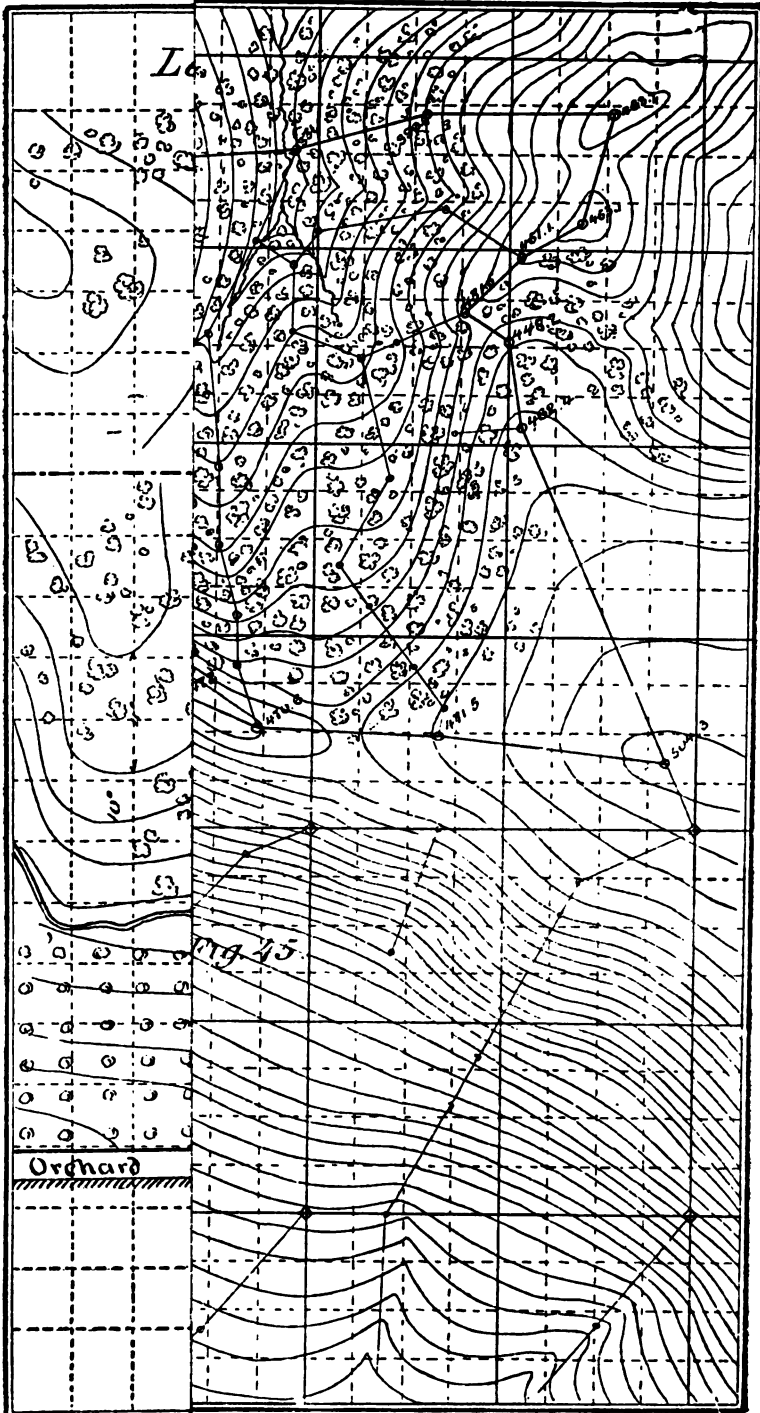
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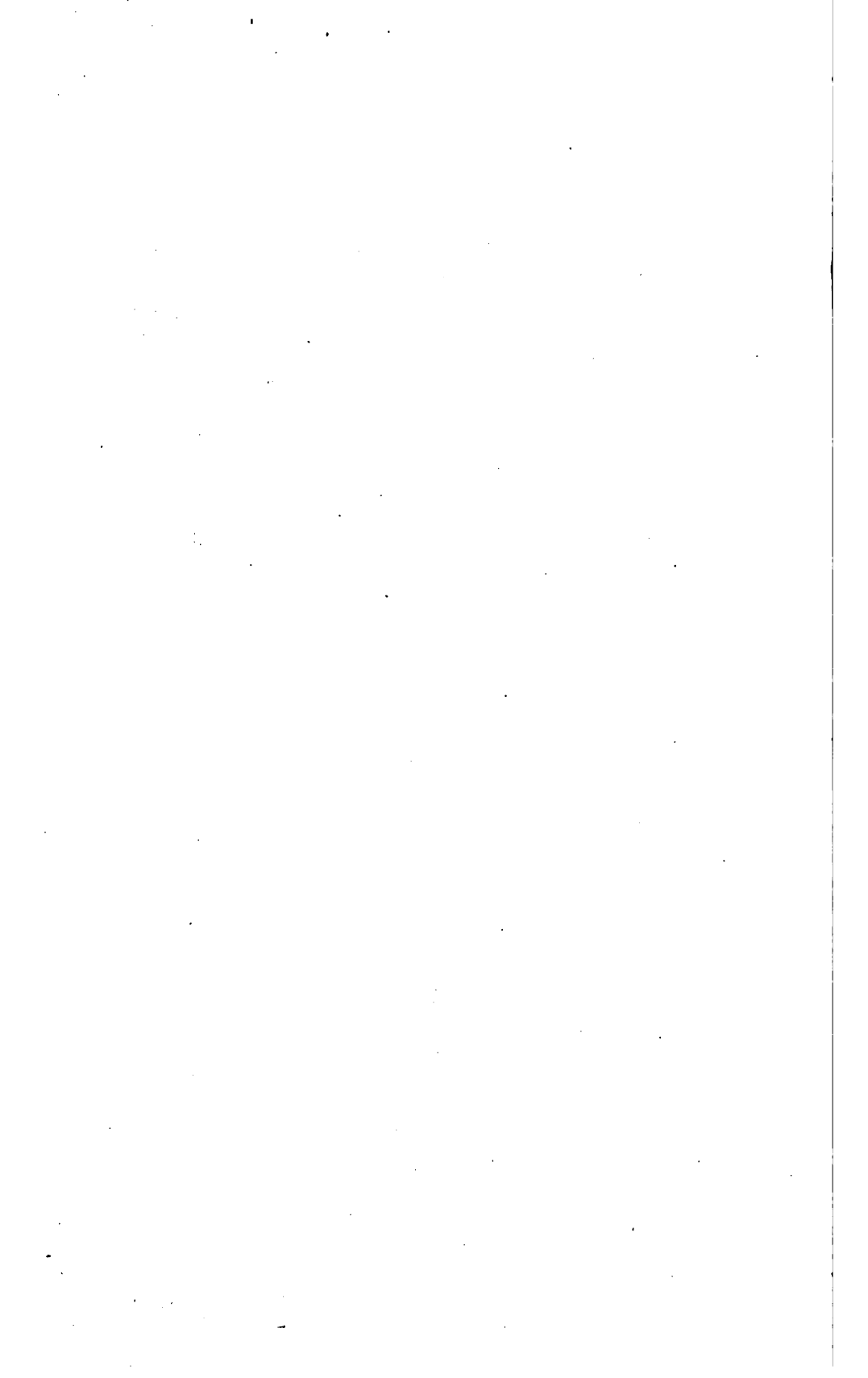
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PLATE VII.





300 400 500

Fig. 48

of construction, $\frac{1}{2} \frac{1}{40}$

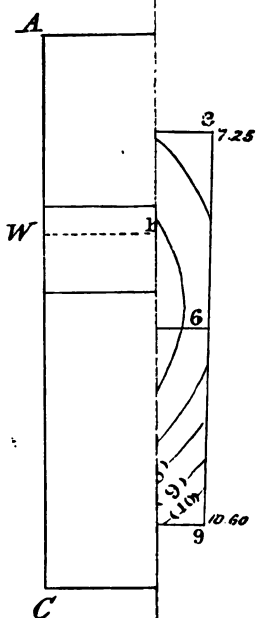
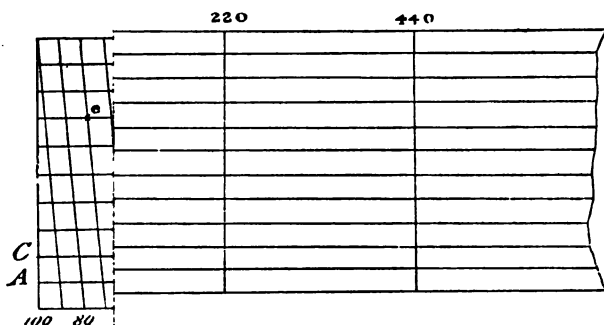
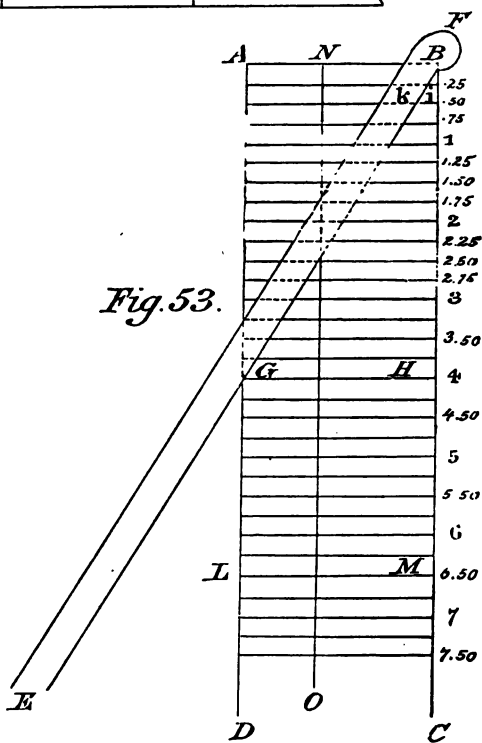
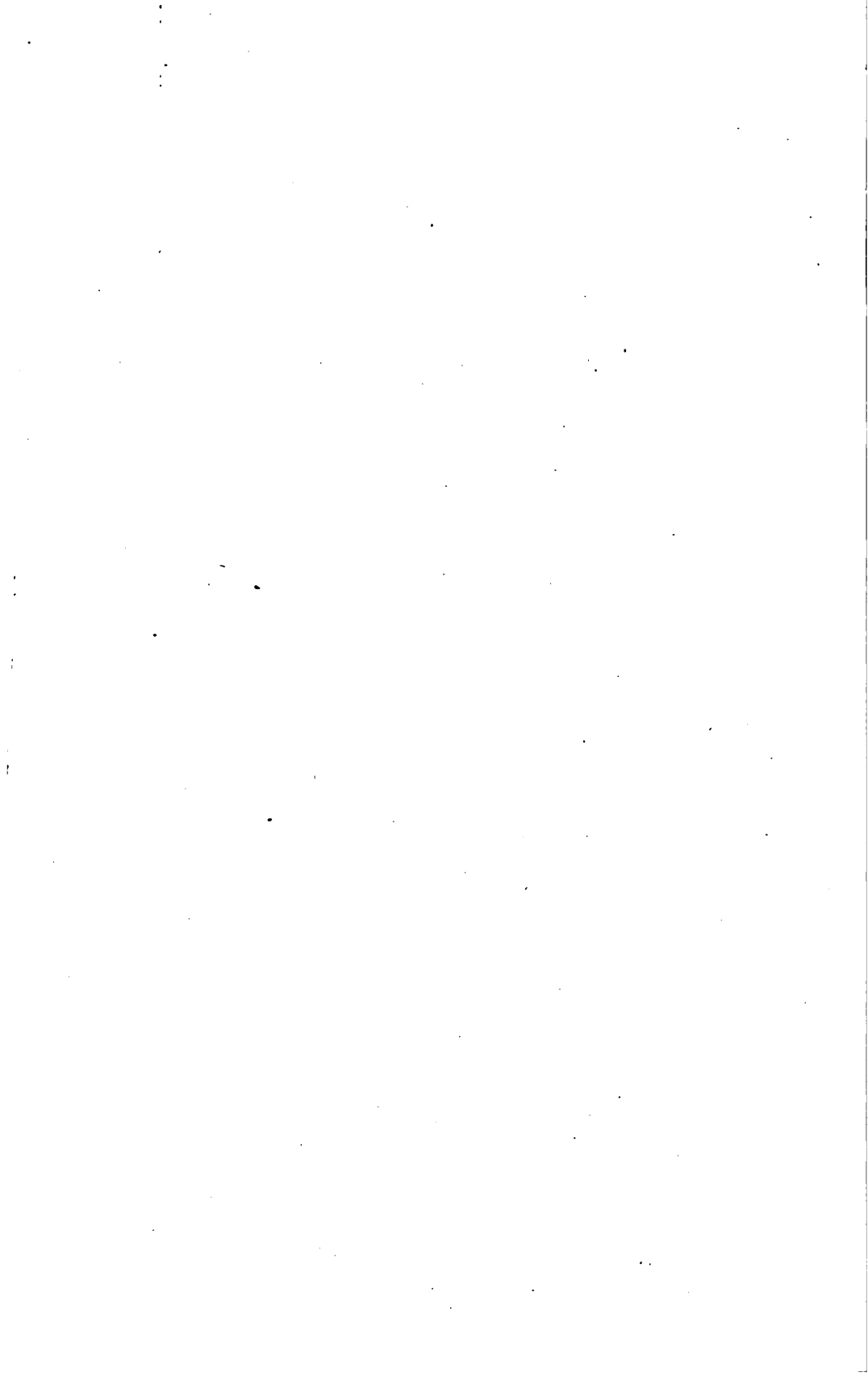


Fig. 53.





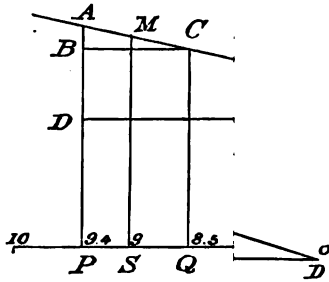


Fig. 62.

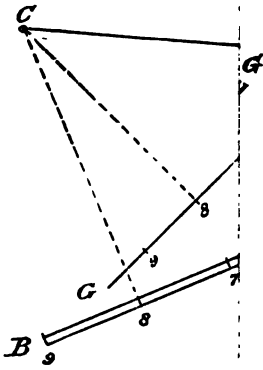


Fig. 63.

Fig. 68.

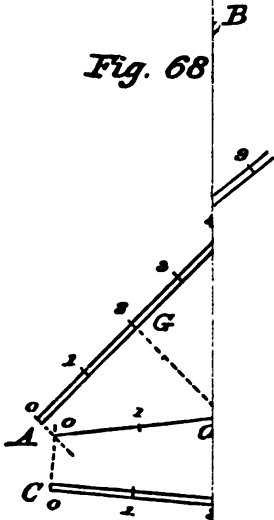


Fig. 64.

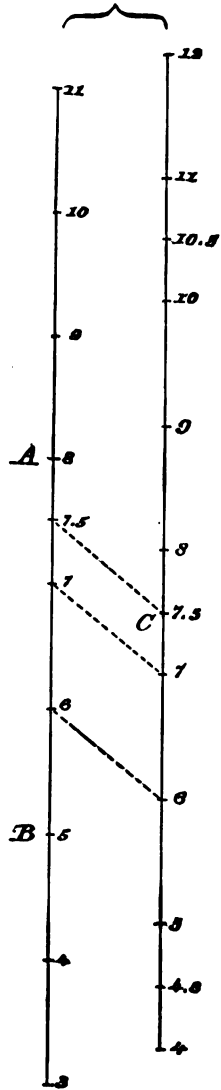
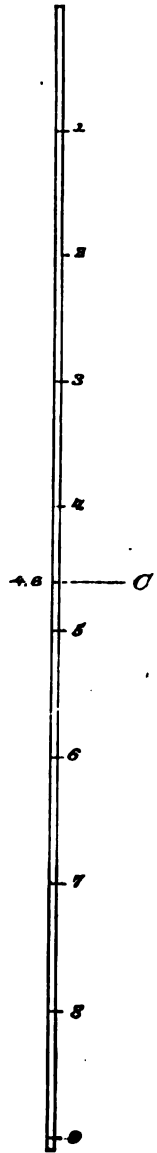


Fig. 65.



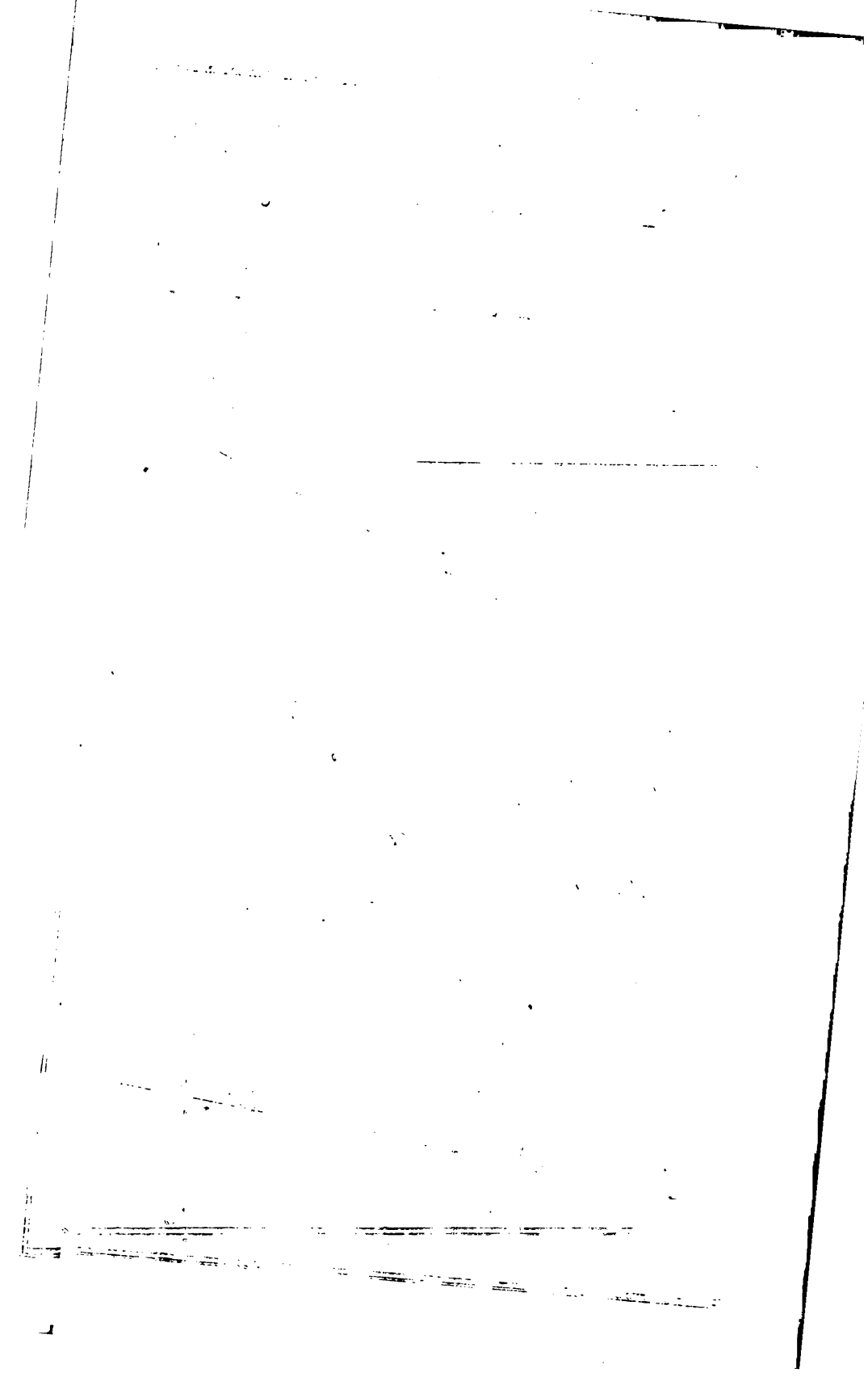
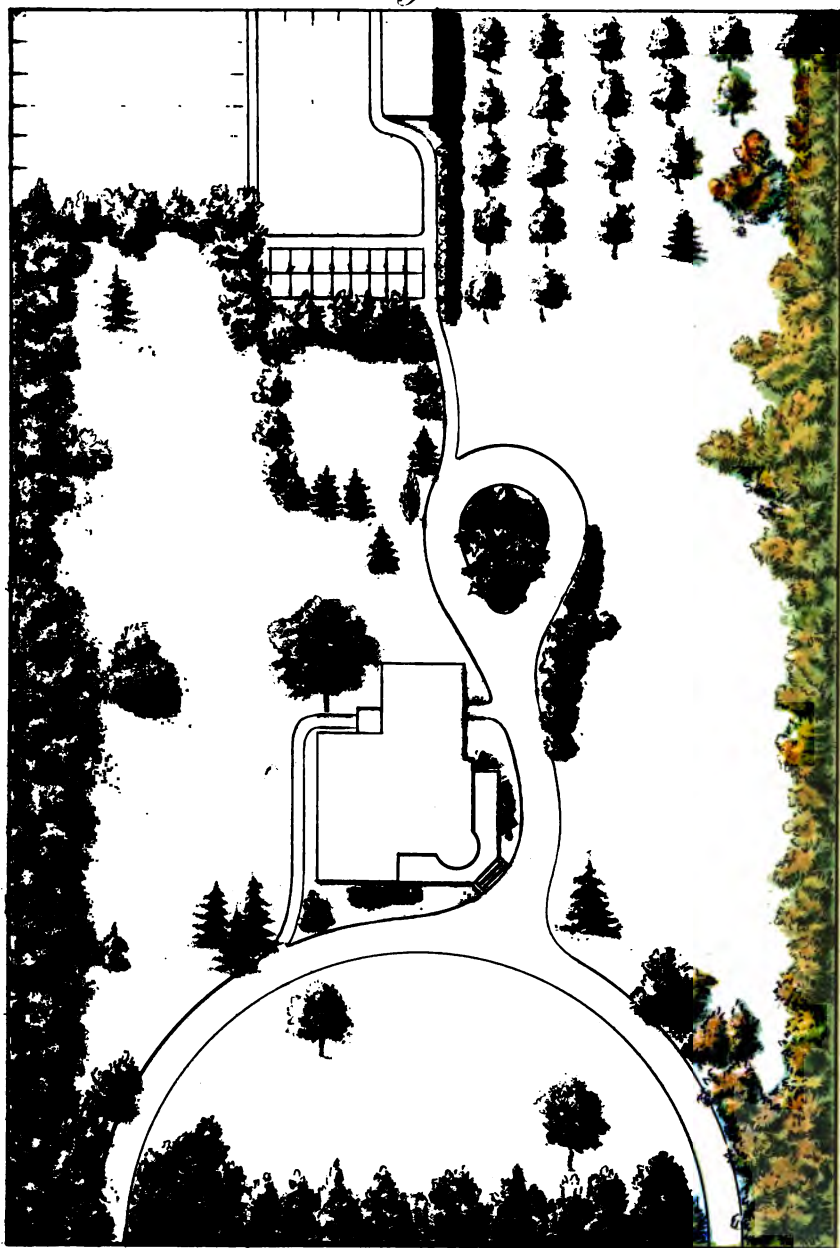


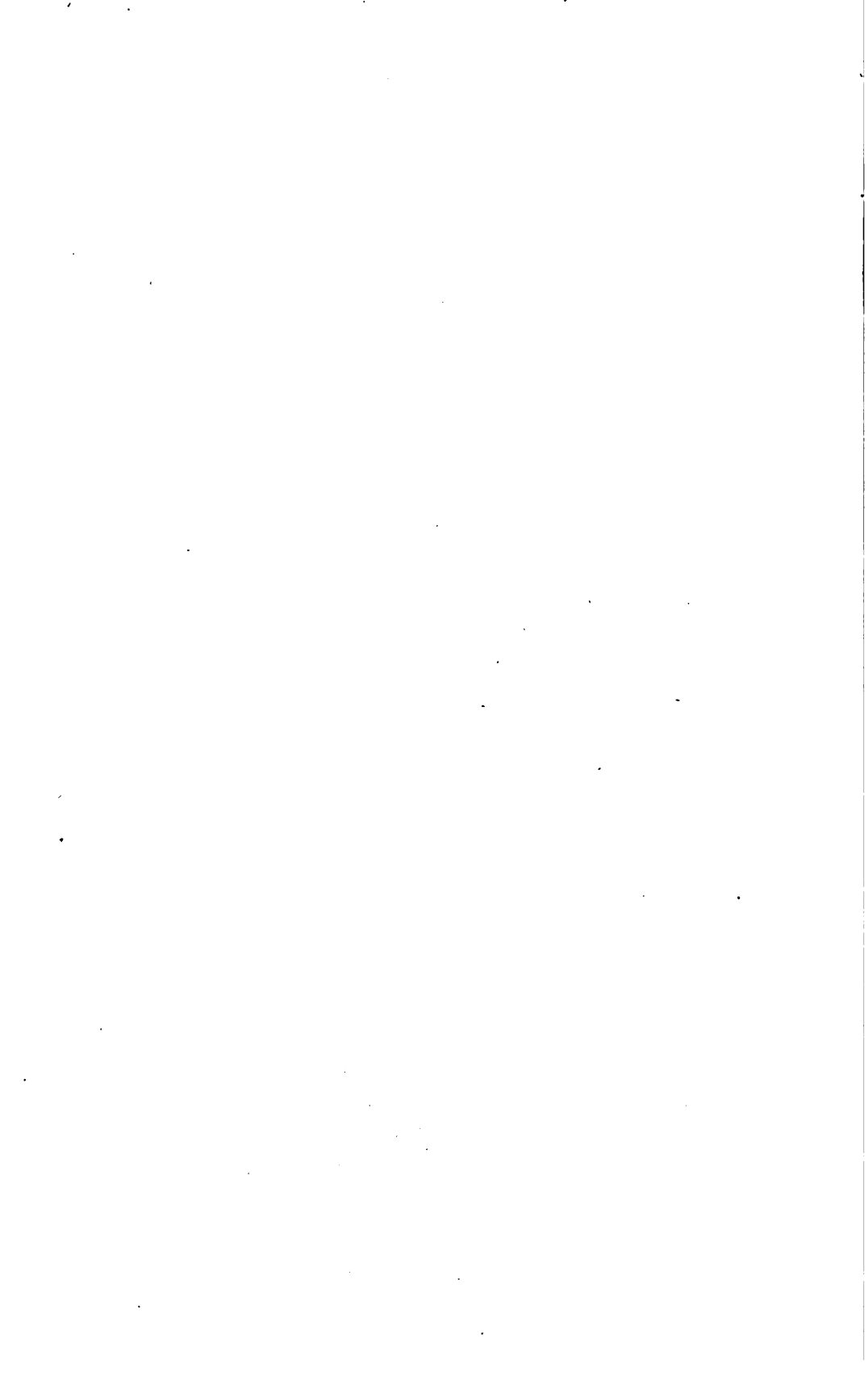


Fig. 70.



Fig. 71.





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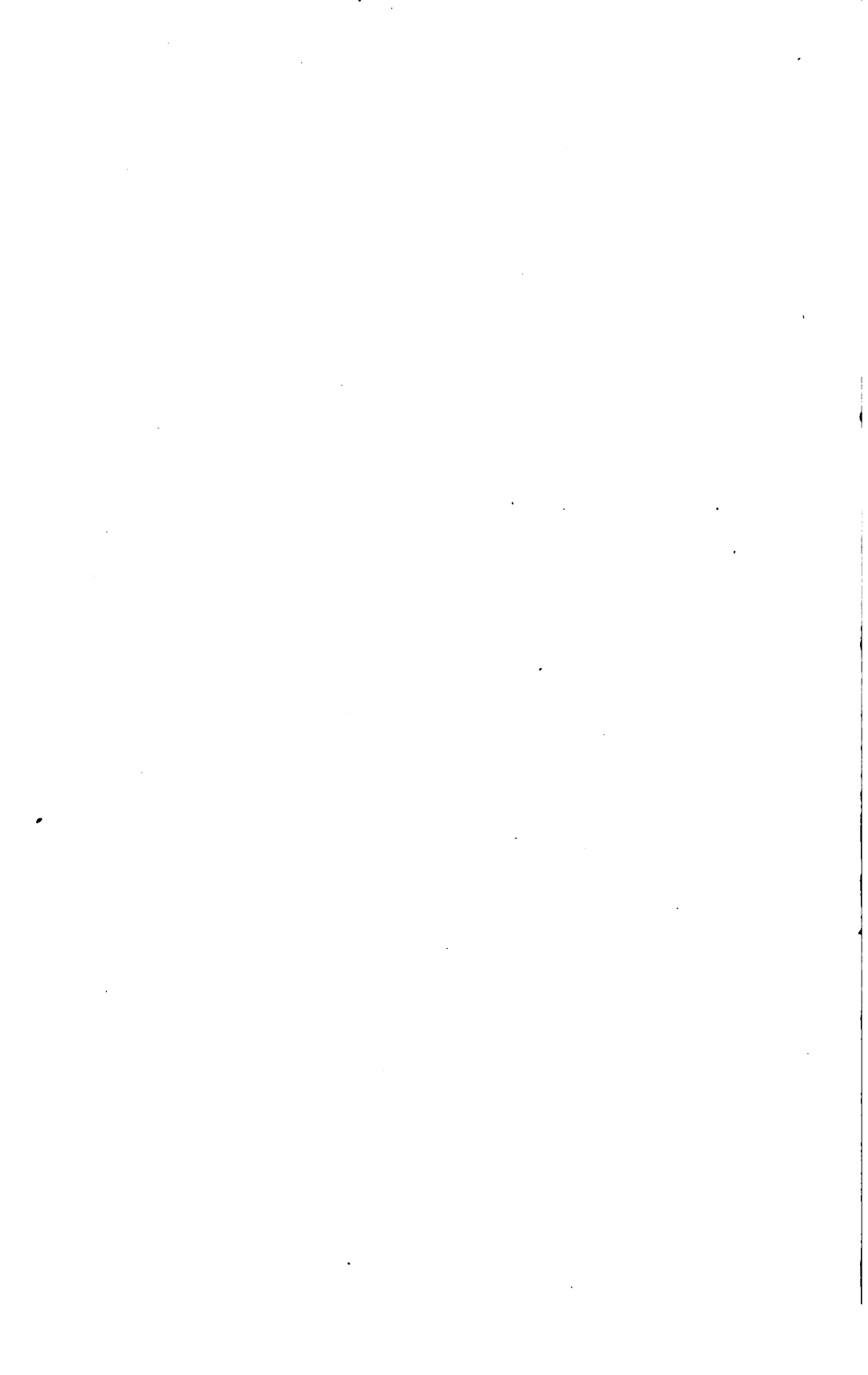
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